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LOGIC AND NATURE

BY THE SAME AUTHOR

Some Modern Conceptions of Natural Law (Cornell Studies in Philosophy)

Cassirer's *Substance and Function and Einstein's Theory of Relativity,* translated by W. C. Swabey and M. C. Swabey.

LOGIC AND NATURE

BY

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PREFACE

A RECENT philosophical writer has remarked that "we are all naturalists now." To this I wish to report one exception. Not that these pages were written to defend a negative thesis. On the contrary, the main purpose of this book is to show that logical considerations form the basis of metaphysics; but, in reflecting upon these ideas, I was drawn on to an attempt to develop a critique of naturalism and an argument in support of rationalism.

It is curious to observe how at the present time certain theories of organic causation and of the biologically instrumental nature of thought are urged against any attempt to give an objective, unpurposive account of being in its general aspects. Yet it is impossible to suppose, on their own assumptions, that these theories of the organism in its relation to the environment are not themselves as organically conditioned as are the theories that they seek to discredit—and their causes as incapable of being discounted in the one case as in the other.

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For, if all human behavior is simply an adjustment to a specifically vital problem posed by the environment, the theory of adaptation must itself be only another such adaptation, and, as such, only a 'way to live,' not a 'science of life' —practical economics perhaps, but not theoretical biology.

Here is a predicament. If thought is essentially a biological instrument, it is itself subject to the biological laws that it seeks to establish. Yet if biology itself is finally determined by the very organic needs and pursuits out of which it is said to arise, its validity as a science may be questioned. For if the study of life is really a reflex of our animal appetites and adjustments, it is obviously not to be taken as a genuine transcript or objective reflection of the nature of the world around us. Biology remains rather a theory of what we as organisms want than of what we and the objects about us are. But unless thought can, in part at least, escape determination by the organism in which it is imprisoned, it remains incapable of achieving the disengagement and impartiality necessary to a science.

Often these difficulties spring from the failure of naturalism to distinguish between *contributing factors* and the *determining conditions* of thought. Whereas it is the merest platitude to assert that our biological makeup has something to do with the character of our thinking, it is the extremest dogmatism to claim that all thought finally expresses nothing but an activity of adjustment on the part of the organism to its surroundings. Nor is the problem to be met by an ambiguous shuffling to and fro from the one position to the other. After all, unless reflection is assumed to be able to stand outside the natural process, and unless it is able to combine or cancel out the peculiarities of behavior of different systems (including its own), knowledge itself becomes impossible. For truth in the last analysis rests in the clarity and nexus of our insights themselves, and not in any dogmatic account of the constitution of our organisms.

How far from adequate this study of logical principles and their applications is, I am most keenly aware. That, apart from the problematic issues involved, inaccuracies have occa-

sionally crept in, is something that, while deeply regretting, I attribute in part to the hazards of any attempt at comprehensiveness. It does not seem necessary to refer individually to the many philosophers from whose work I have derived intellectual profit; in some cases the most helpful have been those with whom I disagreed most. If to some readers who prefer the concrete, qualitative approach to philosophy, my essay seems to fall within the scope of Mr. Santayana's jest against those *esprits forts* who thought the universe composed of nothing but dice and billiard balls (and rigid formal canons), I shall not be surprised. Nevertheless I cannot really believe that those who defend the effable against the ineffable have the weaker side of the argument.

In conclusion, I should like to express my appreciation to Professor Arthur H. Nason, Director of the New York University Press, for his courteous editorial supervision of this work as it was going through the press. I am also indebted to the editor of *The Monist* for permission to reprint that portion of this book which appeared in the form of two articles,

"Reason and Nature" and "The Universe and Universals," in the pages of that journal; and finally my grateful acknowledgments are due to the Open Court Publishing Company for material assistance in the undertaking.

M. C. S.

Washington Square College,
New York University,
February 27, 1930.

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LOGIC AND NATURE

CHAPTER I

EXPERIENCE AND THE LAWS OF THOUGHT

I: The Presence of Change throughout Experience

IF ONE wishes to find out something about the nature of things, a natural place to begin is with an attempt to learn about experience; for, though the realm of experience may not be everything, it is the sphere in which we dwell most familiarly and in which we come to know ourselves as centers of activity struggling with an environment. By experience, I understand the variegated, multiform world which we live through as a panorama of sights, sounds, and shocks. Such a world, although it discloses certain agreeable rhythms and useful recurrences, surprises us on the other hand no less by its harsh non-compliances, vagaries, and blows in the dark. Its shiftings and uncertainties often awaken our despair of ever comprehending it. Yet, in spite of this, we continue to search the labyrinth of phenomena for some guiding thread, in the hope that such a principle, if

found, might transform its trackless confusion into a patterned, orderly world. And it is this hope which leads us in the present chapter to an inquiry as to the nature of experience.

One of the most striking phenomena which the reflective mind encounters in experience is the fact of change. Everywhere, the world is in flux; nothing is at rest or without mutation. From the varying lights and colors of yonder landscape to the vibrating molecules which science tells me constitute the chair on which I sit, or to the incessant vital warfare of the anabolic and katabolic processes of my organism, everything is in motion and is passing into something else. That some objects appear fixed and stable seems to be only an illusion arising from the limitations of the sense-organs. Our senses are too gross to inform us, save retrospectively, of such things as the earth's rotation, of processes of growth and decay, or even of the motion of the hands of a clock. As has well been said, our whole practical life is based upon the illusion that the moving is at rest. Yet if we are to believe the representations of modern physics, the facts are quite other than the ap-

pearances—matter being electrical in its nature and composed of charges vibrating and revolving at enormous rates. Besides being in motion, everything is also undergoing mutation. The vast range of facts bearing on evolution, brought to light since the middle of the last century, has disclosed the transforming influences at work in our world, alterations involving not only the whole of plant and animal life on this planet but extending even to the constitution of astral and atomic systems. All phenomena, these discoveries would seem to indicate, are perpetually undergoing modification and passing into other forms. Thus, in the science of the day, the facts of change and motion occupy the foreground of emphasis and are generally accepted as beyond dispute.

Everything, moreover, appears to be relative to something else. Here, the relativity of the primary and secondary qualities is the most striking example that confronts us; a thing is not extended or colored in the same way to every eye, or even to the same eye at different times. Indeed, nothing can be known as to color and size save by reference to a percipient and a

context. Nor is this relativity of the qualities held to apply merely within the limits of psychology and immediate perception; but it appears with increasing emphasis as a principle of physical science. Not only is the non-absoluteness of the secondary qualities conceded apparently in recent physics, but that of the primary qualities also. Motion, together with the varied modes of space and time (size, velocity, volume, position, direction, etc.), are coming to be admitted to be nothing in themselves, and to subsist only as relations between observers and their chosen systems of reference. But if such thorough-going relativity is true of the realm of experience, if qualities have no locus, and if our world is resolvable into an abstract schema of perspectives, one may well ask, What hope of constancy remains?

A glance at the history of general thought and scientific speculation seems to bear out the point. Everywhere, relativity and flux are traceable. Theories arise and pass away as the expressions of the points of view of individuals and the knowledge of their times. The same facts take on different groupings to different

persons, and are explained in different ways. Just because each hypothesis expresses a specific set of conditions and the perspective of a particular individual, it is always liable to be superseded. Moreover, it is not phenomena alone which change, but the formulas that purport to describe them. One theory is displaced by another, one value transcended by another in unending succession. The history of science and the development of speculative thought read like a melancholy tale of the old being forever cast aside, only to be followed by the new grown outworn in its turn.

The discovery of relativity and process throughout experience suggests certain skeptical conclusions. It suggests, if its truth be granted, that things are not really themselves, are not fixed in their natures, but are always passing into something else and are in their essence indeterminate. Any critical examination of "things" reveals their bewildering complexity. Thus, any object we may select will be found to involve not only an inexhaustible number of factors but interactions with the environment which defy definite delimitation.

Suppose, for instance, we undertake to say what this table is. First of all, we are confronted with the fact that *what the table is* seems to be equivalent to *what it is experienced as* from various points of view. For objects, if we follow the implications of modern physics, psychology, and epistemology, are presumably functions of the observer (that is, of a sensitive recording apparatus or psycho-physical organism which focuses at a given point certain physical perspectives). Yet there may be, for aught we can see, an indefinite number of such observers with experience of the table from different points of view. Accordingly, if we are to be accurate, all possible nuances of size, shape, color, and so on, that may be attributed to the table by different percipients under different conditions, must be accounted part of it. But so manifold and complex is the network of factors that go to make up the table thus conceived, that no limits can be set to its nature. True, we can trace the contributions of a few elements of the "thing" with what appears to be a high degree of probability; but, as no reason can be furnished why a line should be drawn re-

stricting the number of these at any given point, we have to assume the existence of an indeterminate number, though with a constantly diminishing knowledge of their probability as their remoteness increases.

At this point, a contradiction becomes apparent in the nature of the table, due to the fact that "things" claim both to have definite limits and not to have them. In so far as things are objects of exact knowledge and practical action, on the one hand, they are held to have sharp outlines, to be capable of being apprehended in precise, definite ways, and to be reducible to a finite number of elements. Both science and common sense act upon this assumption, treating things as discontinuous with their surroundings and as strictly limited in their factors and causes. On the other hand, science and common sense are forced to concede that the full nature of anything is never actually comprehended within a finite number of elements or completely grasped in any particular case. In sum, nothing seems to have a fixed nature of its own; but everything is a welter of changing factors, a contradictory mixture of the limited

and the unlimited. Yet, if this be so, if everything be ultimately without definite character, we seem forced to the conclusion that the world of experience is a world of flux.

II: The Impossibility of Absolute Change

The preceding section arrived at the view that, since nothing permanent or definite is to be found in phenomena, experience may be regarded as in absolute change. Not in merely relative change be it noted; for, although a world might be partially changing and yet have certain fixed and invariant features as well, everything here is apparently altering in every respect, so that its fluctuation can be described as nothing short of absolute. Yet this conclusion, however inevitable it may appear, only plunges us into new difficulties. For, if everything flows without ceasing, if transition is continuous, how can anything be distinguished from anything else? Such a world seems to offer no genuine foothold for discrimination either as regards *what* changes or *into what* it changes. If everything is in process of becoming something else, nothing appears to be any-

thing at all. Where total mutability reigns, things must be given over to diffuseness and nonentity.

Before accepting this view of experience as involving ultimate confusion, however, it is well to take account of some contrary considerations. Against the evidence offered for absolute change by the apparently limitless flux of phenomena, certain arguments are to be weighed. In the first place, it may be said that absolute change is impossible because it is self-contradictory. Though it be granted that there is phenomenal change throughout experience, this very fact at least remains unvarying: in so far, change cannot be absolute. That everything changes is contradicted by the constancy of change itself.[1] Accordingly, the formula that "everything changes" must be modified to the statement "everything changes—except the law of change." Or, as the phrase goes with

[1] In place of arguing the impossibility of absolute change, a line of reflection might have been developed to show the impossibility of absolute skepticism. But this would have narrowed the standpoint at the outset to the problem of knowledge; whereas the notion of change is broad enough to include all forms of process.

regard to the correlative problem of relativity, *"tout est relatif, voilà le seul principe absolu."* This does not mean, of course, that change takes place according to some particular kind of law, but only that changeless change is persistently so. In brief, recognition of the perpetual flux of phenomena carries with it admission of at least the invariancy of the flux itself.

Another line of reflection calls attention to the fact that change essentially involves its own relativity. Since change can only subsist with respect to some standard taken as unchanging, complete or absolute change is out of the question. In other words, there can be no change unless *something* changes, while this something that makes change possible must itself be permanent. Since nothing comes from nothing, there must be something which does not pass in and out of being but remains indestructible. In this manner, the conservation of substance is sometimes inferred. Recent discussions of the subject, however, deduce the impossibility of absolute change from the notion of a standard of reference implied in change rather than from the category of substance. Relativity to an as-

sumed invariant, it is pointed out, is essential to the nature of change. For the sake of concreteness, let us take *time* as an illustration of our meaning. Whereas it was once believed that all change was change *in* time and not *of* time, which itself remained an unalterable substance, to-day time is held to be relative to the velocity of light. This new standard is declared constant, not by virtue of its substance, which presumably may be undergoing continuous alterations that escape detection, but owing to the fixed ratio and formal invariancy of the relationship between the coördinates of different observers in different systems. Broadly speaking, absolute change appears to be impossible from this point of view, for the reason that the definition of the constituents of any possible world requires the postulation of certain fixed standards and principles of transformation that have to be assumed as logical constants.

III: The Constancy of Change Involves the Law of Identity

But once absolute change is conceded to be impossible, the significance of this conclusion must be recognized. For, if change cannot change, there is something that remains itself, something that is fast and firm in the universe. In short, the fixity of change indicates the operation of logical identity. So long as phenomena shift and flow, flux is flux, or A is A. Perhaps this discovery of identity at the heart of mutability may furnish us a clue as to the nature of experience. For even though there is no phenomenal constancy, it must still remain true that, in the most fleeting and evanescent world, an entity is itself.

But what more specifically is the meaning of this identity? On the one hand, the fixity of change as change testifies to the presence of analytic self-reference: that A is A. In addition, however, there seems to be another, a synthetic kind of identity revealed in our world. We seem to discover identity through apprehending particular changes in experience as examples of this constant universal change. In

other words, we come to recognize the sameness underlying the wayward instability of things not merely by grasping *Change as Change,* but by identifying *particular changes* with *Change as such.* We learn not only that *A* is *A* but that *a* is *A*. That is, in a world which yields analytic identity, synthetic identity (the unification of cases and principles, of particulars and universals) must also be found. Although the law of identity does not, in its usual formulations, explicitly take account of differences (of quantity, etc.) in the terms identified, this very indefiniteness has the advantage of allowing it to cover both synthetic and analytic unities without in any way being prejudicial to its central pronouncement that a thing must be itself prior to its entertaining differences.

It is obvious, if one stops to consider, that such synthetic identity is to be found wherever reflection grasps anything as fixed and invariant. Not only is it impossible to conceive of a specific phenomenal change without involving change in general, but, on the other hand, it is no less impossible to conceive of the latter without reference to particular changes. Moreover,

the very formula of identity (*A* is *A*), although it contains no overt reference to the difference between the particular and the universal, nevertheless implies the presence of such a distinction in the conception of its terms as variables for which any one of a number of specific values may be substituted. On the one side, the abstract principle of identity requires embodiment in a particular instance (that Change is Change, is significant only if I can designate some particular "this" as "Change"); while, on the other, a particular "this" can be conceived as an identity only through joining it to a universal (mutation, process, motion, or what not). Indeed, most of the productive advances in reflection are apparently achieved through this latter generative identity by means of which universal and particular are synthetically joined as law and instance. For it is reasoning that reveals the "this" as being fundamentally an "any"; and the significance of the "any" in its applicability to the "thises." In other words, it is possible to comprehend particular changes as changes only by recognizing their identity with Change as such.

IV: The Constancy of Change Involves the Law of Non-Contradiction

There seems, then, no way of avoiding the principle of identity even in the most changeful world conceivable. And if we would inquire further into the nature of experience, it is again identity that has power to shed light. For, contemplating the progress already made, it must occur to us that change cannot both be change and what is not change. That is, if change *(A)* is itself *(A)*, it cannot be what is not itself *(non-A)*; since, if it could be at once both the same and not the same, both itself and something else, it would apparently cease to be anything definite. By being both change and not-change, it would lose self-reference; since it would violate the law of identity that change cannot change its nature. With the apprehension of this truth, we gain possession of a new law, apparently, in regard to experience, that of non-contradiction, that *A* cannot be both *A* and *non-A*. In its applications, the law is usually taken to refer to the predicates and connections of a term rather than to the identity of the term with itself. For instance, it may be pointed out that

any particular change *(a)* is not merely change, but that it is physical change, simultaneous or successive change; hence, besides being *a*, *a* may be *b*, *c*, or *d* also. The notes of a song as they are sung furnish a concrete example; the fact that they are successive *(b)* is simply a further characterization of the specific type of change (*a*-ness) here; and it would be absurd to claim that the identity of the process of change (*a* as *a*) was destroyed by their successiveness (*b*-ness). On the contrary, the denial of the successive character of musical change would seem to carry with it a denial of the very identity of such change with itself. But, at the same time, the integrity of *a* as *a* forbids that it should be both *b* and *not-b* in any given respect. Thus, the principle of non-contradiction is found to be involved with the law of identity.

V: The Constancy of Change Involves the Law of Excluded Middle

While the discussion so far has brought out that change cannot be both change and not-change or harbor incompatible properties without destroying its identity, the point remains to

be made that this mutual exclusiveness between positive and negative amounts to an exhaustive disjunction between them. This step marks an advance, since it shows that the incompatibility of any pair of opposites is indicative of a complete dichotomy of the universe along these lines. Thus, the very apprehension of identity (that is, of the exact inclusion of change in itself) is equivalent to the definition *per negationem* of all that is excluded from it as a single object of reference (as not-change). But, if what is not included in change includes everything else, we must recognize that between them change and not-change box the compass; hence the acceptance of one is equivalent to the rejection of the other, there being no third possibility left over in the universe.

In the case of *a* and *non-a,* the law of identity (in the form, *a* is *a*) plainly shows which alternative is to be approved and which discarded. When, however, the application of the law has to do with some property of *a* (as to whether *a* is *b* or *a* is *not-b*), the dichotomy remains indecisive in the sense that, although we know that both propositions cannot be accepted, we

usually cannot tell with perfect certainty which should be and which should not. Nevertheless, the discovery of these rational principles amid the fluctuation of phenomena throws light upon the nature of our world. The law that change cannot change is found, when unfolded, to harbor within it the notions of identity, non-contradiction, and excluded middle. The laws of logic thus appear at the basis of experience.

VI: How These Laws Indicate the Presence of the Principle of Sufficient Reason

The disclosure of these laws beneath the fixity of change suggests a good deal that is important as to the constitution of things. In the first place, it reveals that our world is definite and limited in its structure, and, as such, may be grasped by thought and divided into two mutually exclusive and exhaustive portions. Underlying the volatility of experience must be admitted some element of lawful determination; for all particular changes, as we have seen, are essentially definable by reference to the changelessness of change itself, and, hence, are

grounded in the impossibility of absolute change and in the law of identity. Moreover, the discovery of sameness or self-reference beneath the flux illustrates, besides the immediate point of its disclosure, the principle that something is itself because of something else supporting it through which it finds its own nature. In other words, the immutability of the mutable is established through bringing to light the fact that change *within* the process of phenomena is founded upon the unchangeability of the process itself. The detection of invariance beneath the stream of experience is thus made possible through the discovery of the law of ground and consequent. Even the most fluid and variable world has in it apparently some regulatory principle, some condition by which it is possible to pass from certain things to certain others by means of an identity shared among them. And since belief in anything seems ultimately to involve belief in its necessity, the assumption of sufficient reason appears as a condition of our assurance of any determinate realm whatsoever.

The law of ground and consequent implied in experience can be stated in a number of ways.

In the simplest terms, it declares that whatever is has a reason or explanation; and that nothing is true or real which is without logical basis or evidence for its validity. Indeed, the presence of a single reason in the world of experience would seem to be enough to establish the operation of an exceptionless dependency; this being the peculiarity of reason, that a single case, if granted, introduces all others as its support. "For if," said Lotze, "no reason can be given for a reason, what would be its force?" That is to say, no ground *is* a ground unless it has another at its back; yet the chain, once started, cannot break down at any point without incurring the destruction of the entire rational nexus.

VII: What These Principles Indicate as to the Nature of Experience

Even in the most shifting and indefinable state of affairs, therefore, some point of anchorage is to be found; since, wherever change is change, the fundamental forms of reasoning are present also. Were this not so, not only would knowledge appear to be meaningless, but noth-

ing would be anything at all; for it seems impossible to posit anything in regard to the nature of experience save on the presupposition of their validity.

Yet our inability to rescind belief in these laws is something more than simply an inescapable feeling, a blind faith in, or resignation to the limitations of experience. We do not accept them in the same way that we resign ourselves to being five and a half feet tall, or to the impossibility of not seeing through a stone wall with the naked eye, or of not desiring food when we are hungry. Nor is the mere *omnipresence* of these principles throughout experience sufficient to account for their authority. For the phenomenal world, as we come to see, is characterized by a variety of ubiquitous features (viz., by spatio-temporal aspects, by relativity to consciousness, and by the categories); hence the mere tendency of certain features to turn up as present in experience, even when declared absent, is not enough to constitute a unique differentia of the logical.

Apparently our acceptance of these fundamental laws is sanctioned by a twofold criterion:

on the one hand, by a positive intellectual insight into their validity; and, on the other, by an apprehension of the impossibility of conceiving their falsity without reaffirming their truth in their very denial. To begin with, it is precisely because I see in a discursive yet synoptic way that definiteness involves exclusion and the suppression of the contradictory, that I accept the principle of non-contradiction. Whatever positive demonstration of the matter there may be, is to be had by the simple process of *thinking through* the law in question. Unlike other *sine qua nons* of experience, these principles are substantiated by the very train of reflection involved in their apprehension. Thus, all that appears necessary to establish the law of identity is to comprehend its meaning thoroughly. The process, put discursively, might run something like this: "Anything, if it is to be anything (i.e., if it is to have any specific nature), cannot be what it is not itself; for, if it could be at once both itself and something else, it would not be anything definite." The remarkable feature of the principle is, as we have said, that the very comprehension of the logical relation entails ac-

knowledgement of its validity. It appears impossible to understand *what* the law is, without, in the course of the process, seeing *why it must be what it is,* that is, without accepting the intellectual operation involved in the insight as establishing its verity. And were it for some reason impossible to show by drawing out appropriate grounds and consequents what is implied in *anything's being itself,* there would seem to be no other way of confirming the necessity of the law.

On the other hand, such laws also enjoy a negative power of self-substantiation, in that their truth is admitted even in their attempted denial. Suppose, for instance, it is asserted that "The law of non-contradiction is false." Clearly, such a statement derives whatever meaning it has from the assumption that "law of non-contradiction" means "law of non-contradiction" and not something else, and that assertion of its falsity is equivalent to a denial of its truth (or non-falsity). In other words, the declaration derives its significance from a tacit assumption of the truth of the very law declared untrue. Apparently the attempt is hopeless to formu-

late a genuine denial of such a law; since, in order to reason skeptically *about* it, we have, as rational beings, also to reason *in accordance with it* as a principle of proof. Moreover, the effort to doubt any one of these basic relations entangles us not only in its own presupposition but in that of others also; so that, in the end, we are compelled either to forego our doubt altogether or else utterly to renounce reasoning and its canons. If, for example, I disbelieve that "Some non-existent entities are changeless" and maintain that this is false while, at the same time, nevertheless accepting as true the proposition that "No changeless things exist," I may find to my surprise that a whole body of logical relations, like identity and excluded middle, have been challenged by my repudiation of what appeared to be a single statement. It is as if the very mechanism of doubt concealed a trick for its suppression, since the doubting itself sets in motion our use and acceptance of a whole corpus of logical principles. Owing to the peculiar character of these laws, assent to a single one of them seems to have an import far beyond itself and embracing others.

Not only has a law, such as identity, the capacity to shed light in many widely varied and not *prima facie* evident connections, but, in addition, it is supported by a constellation of affiliated principles.

But the charge is sure to be made that this self-substantiation of logical laws takes place by a circular process. The law of identity, it will be said, is already assumed in any attempt to formulate it. For, in reasoning about these most general relationships, it is necessary to take for granted the very things that you are trying to prove. And, in a sense, even the most ardent rationalist must admit that this is so. In reply, however, he might point out that, in regard to certain basic rules, thought is faced with a dilemma: to be either circular or inconsistent. Even supposing *(per impossible)* that thought could divest itself of the laws of thought and could devise a scheme of discourse based upon the assumption of their contradictory opposites, even so, the rationalist would urge, such a scheme, being absolutely inconsistent, would be totally meaningless and unintelligible. Hence, we must accept the other horn

and admit that, in being rational, thought may be "circular"; or, as he would prefer to phrase it, "systematic." For rational procedure, it would seem, is not necessarily circular in a vicious sense, even when dealing with its own basic canons. There is a vast difference, often overlooked, between employing a law of logic as a principle of proof for itself and using it merely provisionally as part of the content of the demonstration. In the process of thinking that leads the mind to the insight that, for instance, any term must be itself (that A must be A), it is true that each particular term is taken to be just what it is in advance of any proof that this holds universally. But, although each particular *"a"* is assumed to mean *"a,"* just as, in thinking, "law" means "law" and "is" means "is," it is only when we come to see that this holds not merely as regards the content of these specific terms, but formally and necessarily for all terms, irrespective of their content, that we reach the insight that definiteness by its very nature requires exclusion; and that this not merely is, but must be, so.

Of course, in a way, all deduction consists in

elucidation; that is, in taking out in the conclusion what was put in in the original assumptions. Yet what is taken at the outset only accidentally or piecemeal as particular matters of fact, is later welded into a new synthetic combination expressive of a necessary whole. The charge of vicious circularity may thus be met by showing that the apodictic connection disclosed in the conclusion was not really present in the premises taken separately or even as an aggregate. When a geometer wishes to prove that a triangle which has two angles equal must also have two sides equal, it is true that he begins by assuming a figure possessed of the very properties the presence of which he is later at so much pains to demonstrate. Nevertheless, in the final deductive insight in which he sees that any triangles possessed of the first property must by their very nature be possessed of the second also, he becomes aware that what previously appeared as only a casual, irrelevant feature of the content, has turned out to be a universal, necessary aspect of the form.

VIII: In What Sense These Logical Principles Are Independent of Experience

If, then, our account be correct, the most anarchic and elusive world of experience cannot be wholly devoid of definiteness and articulation by logical principles; for wherever experience tries to evade these laws, the attempt is thwarted by a tacit appeal to them. Ultimately, the shifting material is subordinated to logical forms.

But, although our discussion has broadly implied the independence of logical laws from the world of experience, this is understandable in several ways. In the first place, the independence of principles may mean simply that certain things are held to be *permanent* amid the wayward flood of appearances and to persist in spite of flux. Again, the distinction between sensuous objects and the non-sensuous meanings or class-concepts to which they point or refer, may give rise to a second meaning of independence. Principles may be said to be independent when they are referred to subsistence in another world than the sensible—to a separ-

ate, transcendent realm of non-sensuous essences. Finally, as it seems to us, the true independence of principles may be found to consist in the self-dependence of logical forms. Possessed of this sort would be such systems as have their essence neither in sensible existence nor in abstract conception, but in their purely rational validity. A simple operation such, for example, as $A < B$, $B < C$; *therefore* $A < C$, contains a pattern that sustains itself through its inherent interrelationships. Behind and supporting the realms of particular objects and sensible appearances to which diction can directly refer, we seem to catch glimpses from time to time of another region that can be only indirectly discoursed about, the realm of the dianoetic processes themselves: forms, operations, systems, which appear to hold for different meanings yet are not to be identified with one particular meaning. And nowhere but in this region of the self-sustaining patterns, as it seems to us, is complete independence of the world of experience to be found. For, so long as independence is defined only negatively as freedom from change or as separateness from the sensi-

ble, it does not signify self-possession. Only in this last signification, however, is independence truly comprehended as self-dependence, as a property of rational forms.

In the end, our brief study of experience has led to certain conclusions, albeit largely negative. The world of experience is a world of flux; its phenomena are particulars set outside each other; their collocations casual, adventitious, and conflicting. Because of its fluidity and contradictions, experience can hardly claim to be the ultimate nature of reality. Indeed, if the facts of experience were all, the universe would seem to have no nature, properly speaking. Underlying them, however, appear certain fixed logical principles.

CHAPTER II

THE GENERAL NATURE OF REASON

I: The Naturalistic Interpretation

THE relation of reason to its natural setting, of knowing to the physical world, furnishes philosophy with one of its most difficult problems. To begin with, reason may be regarded from two different points of view. On the one hand, it may be considered simply as one of several natural capacities (or as a function of them) marking by its presence a tendency of the organism toward abstraction, comparison, and reflection. As such, it may be taken as standing on a level with sensation, feeling, or will, and as differentiated from them only in degree, by virtue of its superior organizing and synoptic power. Or, on the other hand, reason may be regarded from a non-naturalistic standpoint as something preëminent and unique, as a capacity qualitatively distinct from, and authoritative over, the special aptitudes, and as lending man his peculiar supremacy over nature. Reserving discussion of

this latter view for the time being, let us begin with an examination of reason as naturalistically conceived.

The developments of modern science seem to have shown, at least to the empiricist's satisfaction, that man's mind no less than his body is wholly of animal extraction and a part of the world of nature. If this be assumed, the same great evolutionary processes and laws (physical, chemical, biological) that account for the rest of the physical world are held to explain exhaustively the constitution of man. Just as animal life is definable in terms of its physical and chemical constituents, hereditary tendencies, and environmental situation, so likewise human activities, including knowledge, are held to be fully interpretable as functions of these same conditions. Accordingly, from this point of view, what appears to be the mind's free selectivity, its power of abstracting certain features from the stream of experience, and of noting their likenesses and differences, is really nothing more than a mechanical response of the organism to its physical surroundings. In other words, the organism is said to abstract or

select just those features of the total situation which impinge upon it with greatest strength and intensity; thus, we inevitably react to the brightest colors and the loudest sounds, or, at any rate, to whatever stirs most strongly our organic needs and impulses. What looks at first sight like a process of self-determination in thinking, turns out to be only, broadly speaking, a mechanism of natural selection; and all of man's so-called intellectually creative and constructive "action" proves, on closer inspection, to resolve into so many kinds of "reaction." The tendency to irritability, to motor response, apparently constitutes, from this standpoint, the fundamental differentia of life in all its varying forms. At the basis of life, we are told, stands the class of substances known as protoplasm, which are extremely unstable compounds. Given these unstable compounds of C, H, O, N, P, S, and so forth, with their propensity to variation, then, by an inevitable process of interaction with the environment, certain of them break down; whereas others, better adapted to the surrounding physico-chemical conditions, persist and win relative equilibrium

and stability. This tendency of compounds to maintain their equilibrium as against their surroundings, their "inertia" or resistance to change, comes to be distinguished at the organic level as a definite propensity of things to "persist in their own being" *(conatus essendi)* or as a specific impulse to self-preservation. Accordingly, all the actions of living beings are to be interpreted, in this view, not merely as physical and chemical reactions but also specifically as "saving reactions," as mechanisms directed upon the preservation of protoplasm as protoplasm, of life as life.

But if this account be correct in its essentials, the reasoning and intellectual life of man are nothing but so many determinate resultants of physico-chemical laws. Even the most complicated activities of the most highly developed nervous systems have to be accounted for purely in mechanistic terms. Of course, owing to the incompleteness of scientific knowledge at the present time regarding the processes involved, wide differences in emphasis and detail are to be expected among empirical explanations. In certain quarters, for instance, chief

stress is laid upon the concept of the reflex arc as the unit of functional activity; and all higher manifestations of organisms are reduced to the compounding of such arcs. Other hardy empiricists prefer to dilate especially upon recent discoveries connected with the ductless glands and the astonishing variations in psychic life apparently attributable to their over-development or atrophy. Through this means, they find a way to explain man's reflective activities in terms of the chemistry of the body and its internal secretions. Still others incline to stress photo-chemical changes known in lower forms of life as "tropisms." The turning of the sunflower to the sun or the flight of the moth to the candle they take as expressive of the essential principle of behavior-reaction to which man in the highest reaches of his thought must conform. But since, despite various unbridged gaps and divergences in detail, these hypotheses agree in holding all activity to be explicable as a conditioned response to a particular stimulus, they are obliged to construe rational knowledge also (as a form of activity) as the expression of a positive or negative reaction on the part of

an organism toward some definite physical obstacle or incitement in the environment.

II: The Impossibility of Universal Knowledge from the Standpoint of Naturalism

From this point of view, man's most magnificent inferences, his dreams and theories, his scientific laws, his engineering feats, and modification of species, have all to be interpreted as so many complex adumbrations of fundamental organic needs, as ingenious adaptations or working-devices accidentally evolved in the struggle for existence. In the last analysis, we are led in fact to believe that the sole abiding worth of man's religious, scientific, and aesthetic constructions is to be found in their contributory function to life as life. And although this extreme conclusion is sometimes overtly disavowed by evolutionary naturalists, the implications of their doctrine are such that nothing else can well be meant. Theories, no less than claws, wings, and tails, are finally evaluated in terms of the generic problems of survival that they help to solve. The reason or intellect, no less than the

leg or arm, is taken as, in structural principle, only another weapon of refined musculature wherewith to wrest subsistence from a recalcitrant world. Upon these assumptions, knowledge is essentially preservative rather than creative, a defence or acquisitive reaction to a particular stimulus rather than an originative enterprise for reshaping the materials of experience to some pattern of the ideal.

But if man and his capacities are wholly part of nature, and if nature is an aggregate of sensuous particulars, then clearly man can claim no genuine knowledge other than of particulars or knowledge that rises above them. Hence, when reflection seems to soar above the world of special *de facto* considerations and to concern itself with cosmic problems as if it were a universal spectator, let us not be deceived, says the empiricist, but let us remember that man is simply an animal like other animals, a chemical compound like other compounds (for there is no element entering into his composition which is not common to the inorganic world), and that as such he is constitutionally oriented, first and last, upon his organic needs and the mainte-

nance of the stability of his physical system. Accordingly, naturalism denies the pretensions of reason to envisage genuinely formal and universal, as opposed to material and particular, objects. Concepts or generic notions are accounted as nothing more than "generalizations"; while theoretical grounds and reasons are denied all efficacy, being construed as idle, compensatory "rationalizations" after the event. The real forces guiding thought and action are held to be those of our physico-chemical constitution; yet, because we are unaware of this control, we often mistakenly attempt to justify our irresistible motor tendencies by conjuring up *post facto* speculative grounds and ideal explanations of them. The plain fact is, according to naturalism, not only that man is unable to envisage true universals, but that (even if he could) he would be incapable of responding to such ideas as a stimulus. Accordingly, he must reconcile himself to being merely the product of certain specific, mechanical conditions; and must comprehend his moments of apparent initiative, spontaneity, and

origination as only the effects of deeper-lying causes in the order of nature.

In the extreme form in which it is here represented, naturalism is open to obvious criticisms. The chief objection to be offered is, of course, that these ends (life for life's sake and the maintenance of the equilibrium of systems, etc.) upon which nature is said to be directed, are themselves metaphysical and teleological constructions of man's reasoning *about* his supposed animalism, chemical constitution, and so on. Here is more than a suggestion of paradox. For naturalism, despite its disclaimers of all but the strictest empirical and scientific accounts of nature, must find that its very position, if systematically adhered to, constitutes a teleological-metaphysical theory about the world which goes far beyond the warrant of direct experience, yet which seemingly must be granted if the results of the sciences are to be construed by it as either trustworthy or significant.

Furthermore, question may well be raised regarding the view of thought as a biological instrument, on the ground that, even if one grant

that it has in certain contexts an instrumental aspect, abstract thinking is so different in kind from the sense organs, from legs and arms, and from other particulars of organic equipment, as to be hardly comparable with them. The chief mark of most biological tools is the fact that they are bound up with the structure and locus of the organism. The leg, for instance, is attached to the body, and where the body cannot go, the legs cannot go; and similarly with the other members. In other words, the organ apparently requires some direct contact with the environment for any experience or knowledge of it. Thus, even in the case of an instrument of distance-reception like sight, the environment has to come to it. That is, specific vibrations must be given off by the object visioned, must be transmitted by the ether, strike the retina, be carried to the optic nerve, and so on; in short, the experience seems to presuppose something like adjacent or contact action through space between the stimulus-object and the responding organ. In thinking, on the other hand, such direct conjunction or overlapping appears unnecessary; and, as a result, questions

of motion from place to place and bodily behavior become far less important. A thinker may presumably sit quietly with closed eyes and conceive events in Betelgeux or what will happen in the year 3,000 A.D., or review the age of reptiles in prehistoric evolution. In such cases, reflection claims to grasp objects in the past, present, or future environment with which the organism has not, and in all probability will never have, any direct sensible contact. In thus prospectively delineating a state of non-experienced experience and retrospectively describing what the world looked like when there was nobody to look at it, mind seemingly assumes its power to transcend the narrow boundaries of direct acquaintance which circumscribe the organism, and to make use of an organon of knowledge distinct from a particular form of bodily behavior. Of course, it may here be flatly objected by some that thought is simply mistaken in its presuppositions; and that, being only as it were a feeble chemical glow of an animal sensorium on a minor planet, it cannot possibly reliably envisage such remote and transcendent objects. Yet, if this be so, and if

reflection can not be trusted to mean what it claims to mean in regard to its basic postulates, the entire structure of knowledge seems threatened, with the result that not merely thought itself but the whole body of its discoveries (including planets, sensoria, organisms, and environments) appear thrown open to question.

Before taking up these difficulties more fully, however, let us note the existence of certain modified forms of naturalism of wider scope than the foregoing. Many of these broader interpretations recognize the genuineness of reason as a distinctive aptitude and, although still denying its supremacy, nevertheless admit its parity with the other functions of mind.

III: Admissions of a More Liberal Naturalism

From the standpoint of a more liberal naturalism, one of the most striking marks of reasoning as compared with other capacities of mind is its range of comprehensiveness coupled with an apparent economy of effort. Thus, the objects of reason do not require full pictorial representation in consciousness as do, for instance,

objects of memory, imagination, or perception. These latter are held to be trustworthy only when they can be presented as particular existences before the mind in considerable detail. Yet, if we credit current psychology, our range of attention is limited to the apprehension of some five or six discrete objects simultaneously; so that we are plainly handicapped in processes like memory, imagination, or perception, by the time and effort required to marshal a small number of presentations on and off the stage of consciousness. Fortunately, however, there is another aptitude that is largely free of this requirement. Reason or understanding has the power to arrive at conclusions regarding its objects without the laborious, time-wasting necessity of grasping them as particular existences and holding them individually before attention. This is because it is able to lay hold of the form or abstract schema of objects as distinct from their particular content. Thus reason with its capacity of representation through the relational structure of ideas rather than their matter, seems to offer quite incredible resources for the enlargement of knowledge; vastly wider

possibilities of synthesis, in fact, than could be won presumably by memory in history or by imagination in art.[1] This greater scope is also, as was said, correlated with greater saving of time and energy. Were it not, indeed, possible for reasoning to dispense with most of the details of presentation in consciousness, we should sit and perish while seeking to arrive at a small number of conclusions.

Another mark of reason duly recorded by a broader naturalism is the propensity to organize data into systems and to disclose interrelationships among objects hitherto apparently disconnected. Indeed, some have even gone so far as to define reasoning as just this tendency to interconnection persistently applied. In conformity with this inclination, understanding never apparently accepts a "fact" off-hand at its face value or takes an isolated judgment as more than provisional; but requires that each shall be confirmed by linkage with other facts and judgments which mutually sustain and sup-

[1] Of course, this is not to deny that rational activity may include and make use of memory and imagination, but only stresses that it can never be identical with them.

port it. My belief, for instance, that "This azalea is red," appeals for tacit corroboration to the body of my sensory judgments, to their power of correcting and supplementing one another, to the verdict of competent witnesses, and so forth. Moreover, each group of consilient judgments seems to lead on to other groups. In the end, although our reasoning presumably never realizes its ideal, which is that of a single all-inclusive system with no grounds outside itself, it is customary to assume that, other things being equal, the more comprehensive a coherent body of judgments is, and the richer in interconnections, the more reliable it is likely to be. No other capacity of mind appears comparable to reasoning in respect of this power of organizing the data of experience on the one hand, and, on the other, of performing the equally valuable negative function of exposing contradictions.

IV: The Ultimate Weakness of Naturalism and the Assumption of Transcendentalism

But even such broad, eclectic descriptions of reason as the foregoing are open to the charges preferred against naturalism, as it seems to me, in so far as they deny the priority of reason to experience. It is not enough to distinguish reasoning from the other capacities merely in degree, or to note its superiority in synoptic and symbolic representation as compared with them. Its supremacy over the rest of mind must also be recognized, together with its power of illuminating the objective order of things. Only by supplementing the foregoing views with a second view of reason, regarded as logically prior to experience and in so far possessed of a supra-natural character, can the contradictions of naturalism be avoided.

The paradox of naturalism rests, if I am not mistaken, in its assumption that the rational mind and its constructions can be wholly included as a finite part within the sphere of nature. For any attempt to explain the mind and reason as the product of a naturalistic process

must tacitly allow the self-refuting assumption that the process described is itself the product of reasoning. In other words, despite itself the intellect comes to be admitted as both the source and product of nature. Nor is the contradiction to be avoided by taking refuge in the distinction between the *facts* of nature and the *theory* about them, and by claiming that only the latter is the mind's creation; for this very distinction is itself a construction of mind. Had not the mind been adequate to comprehend evolution as a theory, we should have no reason whatever to believe in evolution as a process. Moreover, once naturalism can be brought to see that the nature of nature (i.e., its laws and operations) is disclosed only to intelligence, and that our contact with facts is always in a context of theory (admittedly of the mind's creation), it surely cannot deny, in the absence of all negative instances, that nature as the object of thought is inseparable from the activity and construction of thought which reveals it.

Apparently, the only way in which thought can escape the contradiction of conceiving the intellect as both the source and product of evo-

lution is through the frank recognition of the self-transcendent competency of reasoning to raise itself above the natural order and to be the spectator of its own development in experience. But if, on the contrary, we accept the naturalistic account of mind and endeavor to limit our thinking rigidly to these assumptions, the riddle becomes insoluble how so fragile, uncertain, and accidental a phenomenon as intelligence should be qualified to pronounce a verdict or draw credible conclusions regarding the nature of things in general. What warrant can we have of the reliability of the human mind as a criterion, when, by its own admission, it is so deficient and circumscribed in power and extent? What guarantee have we of the strictness of its leading or the veracity of its conclusions?

Broadly speaking, the tenets of naturalism as apparent in present philosophy may be summarized somewhat as follows: First, all knowledge is derived from experience by methods ultimately empirical; and nothing can be said to exist save what is definable in terms of experience. Second, the favorable maintenance of existence and of the stability of systems appears

to be a fundamental tendency of natural processes. Third, the basic order of nature is temporal and causal; and there is no separate order discoverable of logical dependence. Fourth, intellect or reasoning is only a *proper part* of nature, and, as such, is always less than and included within the *whole* of it. Our contention, on the contrary, is that naturalism, wherever it adheres strictly to the implications of these propositions, involves itself in a self-refuting position whereby knowledge itself becomes impossible. The only plausible escape from this predicament, as we see it, is through acceptance of the transcendent competence of reasoning and the recognition of its capacity as an infinite part to encompass the whole, by which means alone the paradox of intellect (its inclusion within the object revealed by it) seems resolvable.

For if, as naturalism maintains, all knowledge is limited to experience, man can hardly claim to arrive at any accurate estimates of universal processes of the world order; since, in the nature of the case, his organism can never wholly traverse or sensibly examine all parts of

the environing totality, or even, for that matter, a single aspect of it.[2] Yet, if this be so, clearly man can have no assurance of the trustworthiness of the second proposition of naturalism, that survival-value is the ultimate scale by which everything is tested. Such an assertion not only

[2] Thus, the propositions "All water is H_2O" or "Ammonia is NH_3" make assertions that go far beyond the empirical evidence of the cases examined, since only a very limited number of samples have actually been analyzed. Clearly, *mere experience* is not entitled to authorize a pronouncement here as to the nature of the *non-experienced* cases.

To this, the empiricist may reply that the proposition means only that "So far as experience has gone, such has been the case; and, therefore, man has an empirically justified tendency to expect that future experiences will resemble past ones."

But that mere experience entitles us to make this kind of generalization involving past and future is precisely what the rationalist questions. Both past and future for the radical empiricist, he maintains, must be constructions from the immediately present "given" of the organism; and, as such, they never fall within the limits of actual experience at any given time. What we call the past, for instance, is really the work of memory, which constantly selects and arranges sensory material in reverse order, daubing it with the light and shade of imaginative emphasis and, in general, creating an extraordinary fiction of *experience as it was never experienced.* Even more obviously, the futures which figure in our predictions are fictions respecting non-existent experiences, since, strictly speaking, we cannot by any twist of interpretation claim *actually to have lived through* future futures.

transcends the scope of empirical verification, but also expresses a metaphysical insight into the nature of the universe, which is precisely the sort of knowledge that naturalism disclaims as impossible. In the third place, when naturalism denies the ultimate reality of the logical order and reduces all processes to those of temporal succession, it conflicts with science and even contradicts its own conclusions in so far as derived from science. For, wherever science establishes an hypothesis regarding nature, it does so by means of a reflective analysis working in reverse order from that of the temporal genesis assumed to hold in the natural process itself. Furthermore, the very formulation of the law or theory seems to imply that it is revealed to a logical spectator or disembodied intelligence which is able in a single *coup d'oeil* to survey the sequence of events in time and space. Acceptance of the evolutionary hypothesis plainly presupposes the power of mind to raise itself above the natural order. That is, unless we suppose the competence of intellect to outflank and encompass the natural process (unless we assume the part as equal to the whole), it appears

impossible to credit as trustworthy this same intellect's account of the world-process including its claim to be itself a product of it. The logical status of intellect must, if genuine knowledge is possible, be assumed to be independent of the conditions of the emergence of mind in the space-time order; since, on the one hand, the primacy and priority of reason must be granted in a logical sense before, on the other, the evolutionary account of its late appearance in phenomenal history becomes credible. Here, as elsewhere in the system of knowledge, the relation of causal sequence may run directly counter to the order of logical dependence.

V: Can the Act of Measuring Be Itself Part of the System Measured?

But naturalism, in its zeal to construe man as wholly part of nature, overlooks apparently both the inconsistencies in its own premises and the presuppositions of scientific method. That the transcendence of mind over nature is tacitly granted by science in its procedure, seems to us something that can hardly be denied, considering the logic of its assumptions. For only by

presuming the adequacy of intellect to embrace the phenomenal course of events, is science able to place confidence in its own results. Were this power of transcendence denied, the belief in uniformity, the belief that the past and future are as they are thought to be—for that matter, the whole of inferred history and scientific hypothesis—would be undermined.

Moreover, scientists themselves are to-day calling attention to the limitations involved in the strictly empirical or observational standpoint. Recent discoveries have emphasized the enormous biological and physical handicaps to which man is subject in laboratory experimentation. Owing to the fact that the scientific observer is always planted in a human body upon a larger planetary body, from both of which he is powerless to detach himself, universality and objectivity can hardly be claimed for a particular set of observations made from a particular standpoint. For where the locus and activity of the observer are themselves part of what is observed, absoluteness can hardly attach to the individual results, which are bound to be colored by naturalistic peculiarities. Introspective

methods in psychology, for instance, and experiments like those of Michelson and Morley in physics would seem to have shown by their negative results the impossibility of determining the behavior or movement of a system by observations within the system.[3]

Yet, at the same time that science to-day recognizes that the observer cannot jump out of his skin in a natural sense, it nevertheless admits that he can stand outside his private viewpoint intellectually. That is, although we remain imprisoned within the confines of our sensorimotor system and chained to its locus as regards our immediate perceptions, we are still able by means of theoretical reckoning and calculation to discount these impedimenta through correlating the standpoints of different observers with one another according to definite rules, so that the laws of nature or ideal relationships disclosed may be freed from dependence upon accidental features of individuality, and hold not merely for one but for a plurality of systems. This is only another way of saying, as I understand it, that science admits the competence of

[3] Cf. pp. 271-272; also 269-270.

reason (although a part) to step outside itself, as it were, and to assess the whole in which it is contained.

On any other assumption, the possibility of obtaining genuinely universal propositions would almost certainly have to be denied; yet science seems to assert just such universal propositions. For instance, any one who affirms that "all bodies gravitate" is himself possessed of a body, and, as such, claims to come under the law that he enunciates. Now, from the standpoint of naturalism, it follows that, if the formulator of a law falls within its scope, the law is open to suspicion. For, since naturalism denies the possibility of transcendence, it can hardly do otherwise—when confronted with a clear case of the inclusion of the *part* which does measuring within the *whole* which is measured —than question the authenticity of the results. Nor is the difficulty to be avoided by saying that Newton or the observer did not mean to include his own body under the principle of gravitation; since, in that case, he did not say what he meant. If, by "all bodies," he did not mean "all bodies" but made an important exception of his

own, then the vaunted universality of the law is unfounded. Similar contradictions are discoverable in the principle of the conservation of energy, so long as it is interpreted on naturalistic assumptions. That is, either the formulation of the principle must aim to cover even the particular amount of energy required in the enunciation of the law (in which case we have an admission of the self-transcendence of the part to include the whole) or else, if not intended to be included, the pretensions of the law to universality are false. And the same thing holds of evolution. Man, when he formulates the proposition that "all life evolves," plainly claims that he himself falls within the process. And what is this but to declare his body, mind, and whole scale of measurement part of the object measured? Yet, if this be so, what credence can be placed in the theory, in the feeble attempt of the creature to embrace the creative process, of the effect to reabsorb the cause? Acceptance of the naturalistic account of evolution with its emphasis upon the tardy, trivial, and casual appearance of mind in the cosmic sequence, seems here in conflict with the assumed priority of rea-

son as a presupposition of scientific knowledge. Our conclusion is, therefore, that so long as mind and reason continue to be used as the master-key to unlock the riddles of nature, unequivocal recognition should be given to the logical supremacy of the instrument over the object and to its adequacy to compass the task.

VI: The Transcendental Interpretation

Only by admitting the transcendental character of reason, its capacity to raise itself above the natural order and to survey the spectacle in which it moves as a spectator, only so can we gain some notion of an organon that might comprehend the universe.

When inquiry is made as to the outstanding features of reflection viewed as a self-transcendent process, the answer is often that it places the significance of everything in its referable and inferable character. By this is meant that nothing is considered purely in itself or on its own account, but that everything is taken as the sign of something else. In other words, the reasoning mind does not regard its object as a bare datum; but, on the contrary, takes what is given

as the representation of something not given, which serves as its evidence or support. The deliverances of sense, imagination, and feeling, on the other hand, take their stand directly upon immediate experience, not sharing the assumption of reflection that data derive their significance from a source outside themselves. In the language of empirical procedure, "everything is precisely what it is given as, and is not to be explained away in terms of something else." So long as we stand within the actual sensuous, emotional, or imaginative experience, it raises no doubts as to the object, but treats it as so much given fact devoid of extraneous implications. Now, although in most of the enterprises of knowledge both rational and empirical factors are so interwoven as to be scarcely separable, it is nevertheless possible to contrast the two in a broad way by equating them with the methods of induction and deduction as ordinarily interpreted. In the one case, description, in the other, explanation, becomes the ideal of knowledge.

Wherever we aim primarily at acquaintance with particulars, and are satisfied to learn about

"some" without knowing about "all" members of a class, the empirical way of looking at things is of the greatest value. Under these circumstances, observation, enumeration of instances, experiment, and practice play an important part. When, however, our intention is rather to obtain universal insights into the nature of orders or groups as wholes, the standpoint of concrete experience becomes insufficient and has to be supplemented by some super-experiential means. Considering the inexhaustibility of our world, the countless number of beings of any kind that one cares to mention (amœbæ, crystals, stars, men) and the impossibility of our encompassing more than a trifling fraction of any given class empirically, it becomes clear that, if we are to claim the right to frame universal laws and to deal with infinite kinds, we must employ some method whereby a limited amount of direct acquaintance can be made to suffice for a more or less adequate theoretical knowledge of the whole.[4] In other words, sooner or later, knowledge is forced to fall back upon deduction, upon the symbolic representation of

[4] Cf. Ch. VII, Sect. IV, for further discussion.

whole by part, and the methods of rationalism.

An illustration from Leibniz may help to enforce the contrast here between the capacity of rational procedure to master an infinite subject-matter and the impotence of empirical procedure to cope with other than a strictly limited material. When, for instance, we consider the series of squares of the natural whole numbers (1, 4, 9, 16, etc.), we may discover by direct examination that the difference between each square and its predecessor is an odd number, and that these differences, when arranged successively, appear to form the progressive series of the odd numbers. On the basis of this knowledge, we are led to expect that, if we take a given member from the series of squares (e.g., 9) and add to it the corresponding number from the series of odd numbers (i.e., 7), the result will be the next higher square in the series of squares (i.e., 16). This expectation, however, is based upon merely empirical considerations; so that, no matter in how many instances we find that it holds good, it still remains possible that, at some further point in the number-series, the correlation will be interrupted. Only

by adopting a rational deductive approach, in place of an empirical inductive one, is it possible to obtain evidence of a universal and necessary connection between the series of squares and that of the odd numbers. Such an approach discards particular numbers with their peculiar properties, and instead takes numbers in general; thus, *n* is conceived as any natural whole number. By means of *n,* the difference between any square and its predecessor may be expressed algebraically as $(n + 1)^2 - n^2$, a difference which is $2n + 1$ or the value of an odd number. Thus, the universal, non-empirical formula $(n + 1)^2 - n^2 = 2n + 1$, shows conclusively that the difference between the squares of any two successive numbers must, in every case, be an odd number; and that a fixed connection subsists between the progression of squares and that of the odd numbers, so that (by means of the formula) the position of any given odd number can be definitely determined with reference to the series of squares. In brief, the totality of squares and that of the odd numbers are shown to be linked together as parts of one system, each side of which can be known

through the other. So long as numbers continued to be treated empirically and individually, no such discovery was possible; and they remained in a merely external, unexplained correspondence. Once an equation expressive of their essential relation was deduced, however, empirical tests involving the multiplication of instances were rendered superfluous. By the analysis of such examples, the incapacity of empirical methods to comprehend a universal object is made plain, together with the striking capacity of deductive procedure to compass an infinite subject-matter.

VII: The Use of Symbols in Reflection

The activity of reason is always, as even naturalism admits, vehicular and symbolic. That is, reflection never treats what is immediately presented as an end to be believed in simply on its own account, but always as a means, a sign, a bearer of something else. Yet, in leading the mind to other things, reflection, in our opinion, is controlled primarily not by the urge of the organism for practical and instinctive satisfactions as naturalism would have us believe, but

rather by the apprehension of objective relations and logical dependencies. To be sure, analysis of the part played by symbols in reasoning is extremely difficult. And from an external, biological point of view, it may well seem that the instruments and counters used by organisms are mere products of associative habit, devoid of efficacy in themselves, which serve merely as physiological cues for the release of motor responses. Nevertheless, the real import of symbols lies apparently much deeper. Were their significative value really fully expressible in terms of the incipient bodily reactions that accompany them, the concept of the symbol would have to be expanded far beyond its present meaning to include all objects that incited motor tendencies. On this assumption, an apple about to be eaten or a lover about to be embraced might be declared as truly symbolic as the red flag or the Morse code. Furthermore, the whole range of psychic life, including that of inarticulate brutes, could be said to make use of them. But surely so wide a usage of the term would, in the end, destroy all distinctive connotation of symbols whatsoever.

If, however, a narrower conception of symbols is demanded, it is to be had, in our opinion, not so much through construing immediate objects as signals of action as through taking them as substitutes, equivalents, or representations of something not given. In this sense, anything that is regarded as typifying an abstract quality or relational whole may presumably be called a symbol. As regards the specific properties of the emblem and that of which it is emblematic, the greatest latitude can apparently prevail. Anything may stand for anything else; and no qualitative similarity need subsist between the two. What possible likeness is traceable between a pictured crescent and the Mohammedan religion or between a five-dollar bill and the goods it buys? Yet these are generally accepted instances of the use of symbols. Nevertheless, despite the seemingly endless variety possible in the content, the actual relationship between the sign and the object signified must remain constant. Indeed, this latter would seem to be a prime requisite of symbols: the essential import of which is, we may say, to show the possibility of instituting substitutions throughout reality.

Even a cursory analysis of the symbolic process suffices to disclose important features which seem to set it apart as a peculiar instrument of intellection. In the first place, symbolic representation always establishes a connection between a sensuous embodiment and a universal, and never merely between two particulars. Though the emblem is clothed as a particular, the object for which it stands is an ideal essence. Thus, in the algebraic expression $x = y; y = z;$ therefore, $x = z,$ the symbols $x,$ $y,$ and z represent any and every possible set of objects capable of equality. "Well and good," opponents of this view may say, however, "but what of a *particular* arrow on a roadside fence which 'points' to a *particular* town, thus inducing in the passing motorist reactions appropriate to the stimulus 'lodging and the way there'? Surely here is a sign that functions between particulars." By way of reply, it must suffice to call attention sharply to the distinction between what serves as a *sign* in the sense of a stimulus or occasion for a response, and a genuine *symbol.* To 'point to' or 'lead on' from one particular to another is by no means the

same as to stand for, perform the functions of, or act in place of, something signified. Whereas this latter, the true symbolic function, involves the incarnation of the non-sensuous in the sensuous and the identity of the individual and the universal, the former bespeaks a mere 'leading' between particular existents.

In every-day, non-technical experience, the extent of this identity between the particular serving as a sign and the universal signified remains more or less vague and indeterminate. Nevertheless, where the idea becomes more explicit, as it does in the regions of exact knowledge, it is found to indicate the capacity for mutual replacement and substitutability, which may be expressed by the term equivalence. This discovery of equivalence or operational identity as subsisting between the sign and what is signified may be said to constitute the second point of importance to be learned about symbolism. Concretely, what it means is that the symbol comes to function as a complete surrogate for the thing symbolized in certain respects: thus, paper money comes to stand for the things it can buy, just as the Greek letter π serves all

the purposes operationally of the mathematical ratio between the circumference and diameter of any circle.

Lastly, the use of exact symbolism seems also to involve the assumption that, for any value of one thing, there must be some corresponding value of another, and that (through sharing in a common universal) the one can be translated into terms of the other. In other words, as we have already said, symbolism presupposes the possibility of instituting substitutions throughout experience. Of course, in different regions of experience, different universals are used to equate the particulars: thus, in economic relations, it may be money; in other connections, horse-power, volts, pints, millimeters, or degrees centigrade. But everywhere that exact symbolism is highly developed, and especially where quantitative methods are perfected, we have the guiding assumption present that the diverse "many" can be translated into terms of some basal unity. The number-series, the space-time continua, even indirectly those of color, sounds, etc.—all indicate how, in different fields, the value of a given datum may be ex-

pressed as a function of another. Within the realm of number, for instance, any member can, by some operation, be represented in terms of any other; thus, $6 = 4200 \div 700 = 100 - 94 = 3 \times 2$, and so on. Again, space and time may be symbolically treated as continua, in which any given part (be it foot or mile, hour or day) can be defined relative to the rest. Even colors, tones, and temperatures, despite their markedly qualitative character, yield to the application of precise symbolic and quantitative methods. Thus, red and violet, though directly incommensurable, can be more or less satisfactorily compared by the ratio of their wave-lengths: red: violet :: 700: 400 micromillimeters. And, after the same fashion, the qualitative series of auditory experiences can be correlated with a corresponding series of physical vibration-numbers. Nor are we content merely with the discovery of many separate continua in different fields of experience, but strive to establish interconnections between these various systems. Thus, the discovery of the mechanical equivalent of heat and the principle of the transformation of energy discloses a constant ratio be-

tween such seemingly disparate things as, for instance, the chemical energy of coal, the heat of water in a boiler, the pressure of steam, and the motion of the piston of a steam engine. In these and similar examples, the results of reflection, in rendering the data of experience directly or indirectly commensurable with one another through assimilating them to some common measure or symbolic scale, seem to point to the presupposition that reality is a functionally related whole. Indeed, the technique of exact symbolism, resting as it does upon the supposition of interchangeable parts, substitutions, and equivalences, would hardly appear explicable on a different assumption.

VIII: The Supremacy of Reason

In conclusion, we may note certain implications as to the status of reason that follow when it is viewed from other than a naturalistic standpoint. Mention was made earlier of the important part played by the principle of sufficient reason in the activity of reason. Yet acceptance of this law as essential obviously carries with it admission of the impossibility that reasoning

should have an absolute beginning or should have the nerve of its process outside itself. That it should take its rise out of nothing or have a groundless inception, would imply that, prior to a certain point, there had been no operation of ground and consequent. But the very attempt to fix upon a point at which the laws of reason are not operative, involves their affirmation at precisely that point; the process being repeated no matter how far back we seek to place their origination. In other words, if the law of sufficient reason holds at all, it must hold without exception; otherwise we should be confronted at one stage or another with the spectacle of an ungrounded ground or an unconsequented consequent.

Here we strike once more upon the peculiarity of reason: that it is impossible to give a wholly naturalistic account of it or to describe its origination from some other source, without assuming at some point or other (or at least attempting) the abrogation of logical principles. Other mental activities and phenomena, to be sure, can be readily conceived as generated from lower orders of being without thereby violating

the basic laws that make determinate order possible. But this is presumably because, in the world of "becoming," "being" and "nothing" are understood rather as relative opposites held in tension than as contradictories, their possible combination being everywhere presupposed. In other words, nothingness (or relative non-existence) may be conceived simply as a state in which the undifferentiated sum of the qualities of being (or existence) are indistinguishably contained. In the case of reasoning, however, we have something that claims by its very nature to be fundamental and inclusive, and which cannot therefore (so long as its claims are recognized) be united with its opposite and be superseded from a more comprehensive point of view. Because reason appears to be the sole standard adequate to reason, it cannot be negatived without contradiction.

What any attempt to derive reason *de novo* from alien processes imports with respect to the physical world, is a denial of the uniformity of nature and the admission of a factor of pure contingency. And, without doubt, many thinkers of the present day are not unfavorably dis-

posed toward such a view. The very meaning of evolution interpreted cosmically, it is often said, precludes the assumption of rigid lawfulness or exactitude in the processes of nature, and substitutes for them the notion of fluent tendencies only partially regular and predictable, which, at most, register a vague drift away from chance and unaccountable variation toward what seems to be greater determinateness. But if nature is only partially determinate in its constitution, and if events cannot be linked with antecedents in ways exemplifying general rules, chance must be recognized as a *positive agency of lawlessness* at work in our world, and is not to be explained away negatively as meaning merely "control by laws unknown." For the interpretation of chance as the equivalent of this latter amounts to no more than the reinstatement of uniformity under the name of its opposite.

Yet, in trying to conceive the scope of order thus restricted, it appears to me, our confidence in our world would be inevitably undermined; for any skepticism as to the prevalence of uniform connections must also cast doubt

upon our judgments about the world to an unascertainable extent. And, though we may still assume that, like the animals, we are driven to action by blind habit and instinct, nevertheless even our reliance on these clues cannot be claimed as matters of knowledge, but rather only as preferred dogmas or ungrounded beliefs. Even if uniformity is only partially denied, we can no longer, as it seems to me, proceed in our daily affairs with an intellectually confident attitude, assuming nature to be a stable, harmonious order of which we can take advantage for our needs and purposes. For facts, on this new supposition, retain the right to swerve unaccountably and to indulge in fits of waywardness and caprice. To a degree at least, it remains possible that anything may happen at any moment. Moreover, chance here becomes a positive force which defies prediction and flies in the teeth of the laws of probability; for, since the power of prediction rests upon the assumption of uniformity of sequence and the calculation of chances in the light of this, it can only be where the laws of combinations and permutations themselves fail to work that we have

the real locus of contingency. And this may be anywhere on such a theory. Once the logic underlying experience is impugned, we have no right to be surprised at the most fantastic capers in nature: at the cow jumping over the moon, at fish careening through the heavens, at an altogether mad and topsy-turvy world.

What those who seek to impose limitations upon reason fail to realize, as it seems to me, is the magnitude of the task that they have set themselves. For if reason be denied, to what higher court of authority can we appeal? True, intuition, imagination, or the senses might seem at first sight to furnish such a tribunal; and, so long as we stood within the circle of one of these experiences, it might presumably raise no doubts. But once we began to compare their deliverances, we should become aware of a hopeless conflict of authorities. On the one hand, the hard, niggardly data of the senses would checkmate the mad fictions of the imagination; while, on the other, contrary gusts of emotion would threaten to tear us asunder. One look below the surface reveals the natural man as a wild beast dragged hither and thither by a storm

of conflicting tendencies. How can peace and order ever be brought to such a world? Only, it would seem, through the rise of some power of criticism; only through the emergence of some impartial authority, like reasoning, able to raise itself above the strife of our many-sided nature and to proclaim itself arbiter and reconciler of opposites.

CHAPTER III

THE PROBLEM OF THE FUNDAMENTAL UNIT OF REASONING

I: Rival Claims of the Concept, the Judgment, and the Inference

SINCE our investigation of experience has led beyond the parade of flux to the discovery of its establishment in rational principles, the question naturally arises as to which among these principles constitutes the basal unit of reasoning and the bed-rock of its structure. At the outset, we are confronted with a very general opinion that laws like identity and non-contradiction, taken by themselves, are too negative and analytic to furnish a dynamic, positive core of the noetic process. Without denying the importance of these laws in defining and delimiting the universe, it is none the less maintained that reason must also operate in some more constructive, concrete manner. Indeed, only on the assumption of some further unit capable of embodying rules and synthesizing data, does it seem possible to account for the

actual organization of truth and advancement of knowledge. Although the laws of thought have their use in the removal of inconsistencies, we must apparently look to another source for an instrument of proof and discovery.

Yet, as regards the nature of this elemental structure of reasoning, opinion is divided. Three rival claimants appear in modern discussions of the subject: the concept, the judgment, and the inference. In the present chapter, it is our purpose to weigh all three and to decide which form is the most fundamental. In connection with the laws of thought, it may here be noted that the *judgment* is sometimes taken as the concrete expression of non-contradiction as a principle of predication, while the *concept,* owing to its fixity in remaining itself, is correlated with the law of identity, and the *inference* with sufficient reason. Indeed, no stretch of imagination is required to see how this latter law, that from certain grounds certain consequents follow with logical necessity, is but an indefinite formulation of the principle of inference.[1] Yet,

[1] Wundt, W., *Logik,* vol. I, 3rd revised ed., p. 299. "Thus the Principle of Ground is first considered to be the most general law of inference."

the bearing of inference upon reasoning, something which at first sight seems very easy to explain, is a matter of dispute. Although it is generally recognized that to reason in the full sense without inferring, or to infer without reasoning, is impossible, it is by no means agreed that to reason and to infer are precisely the same thing. Even where it is not made clear *how* the two are to be distinguished, they are, none the less, not conceded to be identical. An impression often conveyed is that inference is a *sine qua non,* a complex development that emerges in the course of reflection, but that the heart of the process is something simpler than inference and fundamental to it.

An explanation of this sort that is widely received places the essence of reasoning in the judgment, or, less frequently, in the concept, instead of in the inference. Both the former, it is claimed, arise prior to inference in the order of knowledge as well as temporally earlier in the genesis of consciousness, and hence must be more basic. Moreover, any given inference is obtained by the combination of concepts and judgments, and into these components can

again be analyzed. Although, in our developed thinking, we rarely, if ever, reason without inferring (in other words, we never stop short with the mere elements of conceiving or judging, but push through the process by *deriving a conclusion from them*), nevertheless the ultimate constituents of reasoning are said to be located in these simpler units, of which the inference is only the complex resultant. Thus, on the basis of a claim to genetic and epistemological priority, the concept or the judgment or both are held to represent the basic elements of which all reflection is constructed.

But—even if it be granted, for the sake of argument, that in the historical order of events man may conceive or judge before he infers, and even that concepts and judgments are the material or stuff from which inferences are formed—even so, this by no means proves that the former are logically fundamental in the nature of reasoning. For priority in the order of knowledge does not establish logical priority. And, until it is shown that the concept or judgment comes first in order of structural dependence (that is, that it does not de-

pend on the others although they depend on it), neither can be said to be demonstrably the basis of reflection. Yet to prove so much seems well-nigh impossible; for although it is easy to show that the concept and judgment are among the simplest factors that enter into reflection, this simplicity of theirs, being nothing but lack of differentiation or relative structurelessness, in no whit proves their logical primacy. The truly basic element in reasoning need not be devoid of parts, but should rather exhibit a unity that would withstand reduction to terms other than itself.

Now, the view that is advanced in the present chapter, and which it seems advisable to anticipate at this point, is that the inference, and not the judgment or concept, constitutes the ultimate element or type of reflection. Accordingly, the arguments that appear to support the claims of inference against the others may be presented in due order. In the first place, it seems impossible that either of these simpler structures should suffice as the unit of intellection for the reason that each, when taken in isolation, is essentially *de facto* and unfounded.

The detached judgment, for instance, appears by itself as a mere flat, ungrounded statement claiming truth on the strength of its bare assertion without support. And although the judgment may perhaps claim to be a "natural unit" of reflection, it can by no means be shown to be a "necessary form" of it, which alone could validate its title to a primary position in the rational process. How serious is the mistake that identifies the unit of reasoning with one or the other of the simple morphological structures in which thought takes shape, becomes clear when we attempt any description of reflection. To reason, we say, is to advance from the "given" to the "not-given," from the known to the unknown. Yet the distinctiveness of the operation does not subsist in any novelty of its content apparently, but rather in the fact that a synthetic result is achieved through a propulsion attributable neither to inner nor to outer forces, but to the pure generative form of the process itself. Moreover, when we seek to discriminate this compelling element further, obvious reasons occur why it cannot be located in the single concept or judgment. For, although

each of these holds together a manifold as a unity and is in so far synthetic, neither has in itself the capacity to give rise to something new and additional. On the contrary, a detached expression of this sort stands more or less dogmatically on its own fiat, whereas the essence of reflection seems to consist in some dynamic transaction whereby the very interrelations of a manifold generate a valid advance.

Again, if we turn to actual thinking for purposes of illustration, it is readily notable that the concept and the judgment appear too truncated and incomplete as forms to serve as adequate unities of thought. If, for instance, we are confronted with a bare term in abstraction from its context, our immediate reaction is one of confusion and importunate demand for the instatement of the term in an explanatory setting. Thus, if we overhear the bare word "weather," our minds (far from accepting the announcement as sufficient food for reflection) grow at once clamorous with questions: "What about the weather? Is the prediction fair? Did he say 'weather' or 'whether'?" etc. And the same thing happens to a less degree when a

judgment or proposition occurs apart from a relevant context. If you find, say, in a work on logic, the detached statement that "No insects have eight legs," you might expect that, since this expression conveys a definite meaning and modicum of information, it would fairly satisfy our minds. But thought, far from accepting it as it stands, is roused to seek linkages with the rest of knowledge before according it adoption. "Why cannot insects have eight legs? Is this a basic structural distinction? What is the authority for the statement?" In some such manner, the mind is spurred to interrogate by the impulse to complete the proposition's incompleteness. That the bald announcement stares like a glass eye is not because it is denuded of all meaning (as might be the case with a single word) but because of its apparent disconnection from any larger whole that would give it place and status. In dealing with terms or propositions as with facts, it is apparently the nature of reflection not to accept anything at its bare face value, but always to demand 'leadings' or interconnections that shall substantiate and confirm it. Accordingly, the discrete judgment

and concept must both be rejected as the ultimate form of reasoning because of their inadmissable claim to be rational without reasons.

It would seem that any attempt to regard the simple judgment or concept as fundamental does so only by overlooking the principle of ground and consequent, which affirms that every valid proposition must have others on which it rests. To make the discrete judgment ultimate, if I am not mistaken, is thus to ignore the principle of its own cancellation which every judgment carries. As Sigwart puts the matter, "Every judgment claims to have a logical ground which makes it necessary. . . . Judgment cannot take place without a ground, for by the ground we mean just that which makes a judgment necessary."[2] The same thing may apparently be suggested in another way by construing propositions as ultimately modal (i.e., as apodictic or problematic in import) rather than as assertoric in their nature. In other words, the judgment is to be understood in its real meaning as the conclusion of an inference, as the expression of a law rather than as an as-

[2] Sigwart, *Logic*, vol. I, p. 189 (Tr. by H. Dendy).

sertion of existence. Thus, when I say "This triangle has the square of the hypotenuse equal to the sum of the squares of the other two sides," what I mean is "This triangle, being a right-angled triangle, must necessarily have these properties." Similarly, when I say "Some birds sing" or "Most men wounded in the liver die," these propositions express logical relationships that subsist between their subjects (acting as premises) and their predicates (acting as conclusions); i.e., between birds and singing, and being wounded in the liver and death. Also, they express a certain vague probability as to the realization of this connection in the specific case: birds *may* sing, men *may* die: there is a certain likelihood of their doing so. Such propositions may be called problematic, and may be taken as modal in their essence, since they express probability, which—although vague and indecisive as regards the individual case and the actual event—nevertheless requires the supposition of apodictic connections and a logical necessary theory underlying them.

In other words, it is only apparently by exhibiting the relation of subject and predicate in

the proposition as one of ground and consequent, that a necessary connection can be established between them. But this is only another way of saying that the claim to necessary connection made in the judgment or proposition is substantiated only in the inference.

Lastly, what appears to be one of the most significant grounds for believing that inference, and not some simpler form, is alone adequate to constitute the basic type or unit of reasoning, is to be found in the nature of inference itself: in its systematic and self-sustaining character, features in which it differs widely from the preceding, less differentiated structures. The treatment of these prerogatives of inference must, however, be deferred to their proper place in the next chapter. In the meantime, there still remains an imposing obstacle to the comprehension of inference, an obstacle arising from the false interpretation and undue importance ascribed to judgment in contemporary thought, and hence requiring separate treatment.

II: Errors Arising from the View that Judgment Rests Essentially on Suggestions of Sense

Considerations that have lent great weight to the view that judgment is the ultimate form of reasoning, are those that represent judgment as the peculiar activity that bridges the gap between the purely intelligible world and the world of sense. Only after judgment has performed this synthesis, it is thought, can inference properly be said to take place. In descriptions of judgment of this sort, reasoning as a purely theoretical activity plays often relatively little part; the mind's selection of, and response to, the data of the environment appear almost as a function of the data themselves. The judgmental act is depicted as occurring upon the occasion of some immediate sensuous contact, to which the mind responds by abstracting and uniting just those elements of the total impression which force themselves upon it most insistently. Here the mind, instead of being guided by its capacity of logical self-determination, is constrained to yield to the compulsion of the data, its sole distinctive act being the accept-

ance and gathering up, as it were, of precisely those elements rendered irresistible by what, in a broad sense, may be called the forces of natural selection operative upon attention.

The significance of such views of judgment for the rest of knowledge can hardly be overlooked. Either they imply that pure reason must be banished altogether, or else they presuppose that there subsist two distinct and equally basic worlds, the one capable of being felt, but not known theoretically, the other capable of being known, but not sensible. Outside of philosophy, it may be remarked, the tendency to deny reasoning any autonomous activity and to assimilate its behavior wholly to that of the stimulus-response mechanisms of lower organisms proceeds without abatement at the present time. Inside of philosophy, however, it still appears impossible (despite many wavering attempts) to get along without assuming the integrity and efficacy of intellectual procedure or without appealing to the validity of its results in a logical, as distinct from a biological, sense. But, as a result, present philosophical thought tends to be dualistic—

recognizing, on the one hand, a world of demonstrable reasoning; and, on the other, an indemonstrable reality with which we are made acquainted in sensuous contacts. In the judgment, many believe, these two realms are somehow brought together and achieve a synthesis; since, in the judgmental act, abstract reflection seems to grasp and incorporate elements that are in themselves irreducible to theoretic terms.

The whole issue presumably turns on the question whether knowledge is the attempt to unite two alien worlds. If it be true, some would say, that we encounter a certain obdurate and impenetrable core in reality, then the judgment may well be accepted as the primary, originative act of intellection. For judgments appear to combine in a rudimentary way both susceptibility to the hard data of sense and a power of intellectual synthesis; and thus seem peculiarly fitted to play the *rôle* of intermediary or connecting link in knowledge between abstractions and immediate sensa. On the other hand, in our opinion, the very dualism of such a view renders it inadmissible. So long as thought is represented as confronted by an alien element

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in reality, by something unassimilable and intransigent, so long it seems to us impossible to discover any theory of judgment under these conditions that shall successfully bridge the gap between two worlds. At best, reasoning can be said only to weave a web of interpretation about this inscrutable, underived reality which is everywhere encountered without being apprehended; so that the function of judgment remains, in the words of one of the distinguished exponents of this view, that of 'asserting the Unknown.'[3]

But enough has presumably been said to indicate that, in our view, nothing can be made an object of reasoning without being reduced to terms of it; its peculiarity being to allow no ultimate 'other' over against itself and to admit nothing into itself that is irreducible in kind. Hence, unless we can assume that there is an unbridged gulf between the worlds of sense and reason, or unless we can banish reason altogether—both unacceptable hypotheses, it would seem—we must believe that the world of the

[3] Bradley, F. H., *Appearance and Reality*, 2nd. ed., p. 362.

former is, in principle, indefinitely assimilable to terms of the latter.

III: Are Universal Judgments Hypothetical?

A further unfortunate consequence that apparently follows from interpreting judgment as a response to sensuous stimuli is that it becomes necessary to consider practically all universal judgments as hypothetical in character. Although this consequence is not always perceived and admitted, it is, as we shall seek to show, an inevitable corollary of the view in question. For where judgment is conceived as rooted in an immediate contact with sensuous reality, it is obviously conditioned by the sense-organs of a responding organism; so that the question properly arises as to *whose* perceptions or impressions are actually expressed in any given judgment. For instance, if I judge that "Oranges are yellow," is the proposition meant to record my own visual experience? Apparently not. Indeed, if I am a blind man, it may not describe my own experience at all. On the contrary, it is said, what is asserted in the or-

dinary universal judgment is not the perception of any specific person, but rather the experience of *sense perception as such* indefinitely extended and freed from the limitations which govern the particular individual. Yet, plainly, we know of no such powers of perception-in-general as actually existing. The only way, therefore, in which the empiricist is able to retain his thesis that knowledge is rooted in direct contact with the sensuously given, while at the same time admitting that no such actual contact in many cases exists, is by construing universal judgments hypothetically. That "oranges are yellow" is said to signify that: "If there were an observer with infinitely enlarged sense-organs and powers of perception who could perceive oranges wherever they were, he would everywhere take them to be yellow," or, otherwise stated, "If oranges were under the proper conditions combined with the presence of a spectator, they would yield to that spectator the perception yellow at any place and time."

The hypothetical interpretation of universal judgments is of particular interest, as we shall see later on, in connection with the judgments

of science. While natural science is, on the one side, strongly empirical and seemingly in accord with the view that true knowledge must be grounded in direct observation and experience, on the other side, its theoretical tendency urges it no less strongly toward the formulation of universal laws, which almost invariably outrun the empirical evidence in their claims for the prediction and control of nature. This apparent inconsistency in the standpoint of science is held by many thinkers to be solved through recognition of the essentially hypothetical character of scientific laws. For the only way, apparently, in which the tendency of thought to transcend the limits of actual experience can be accounted for in a manner at all congruous with an empirical basis of judgment, is by construing universal propositions as if they were the judgments of an ideal whole of possible experience. Although finite experience is in its very nature incomplete, being confined by the barriers of the organism, it is perhaps possible to imagine a *mögliche Erfahrung,* or hypothetical unity of what the totality of objects could be perceived as, from all points of view, that

should be complete and free from all restrictions. Despite the fact that we know of no such actually existing observer, the postulation of one seems to many an epistemological necessity. And since, by definition, such a supposed Whole of concrete intuition would experience all-there-was-to-be-experienced, its judgments would express universal propositions or laws of nature. Hence, the aim of science in formulating laws is said to be nothing more nor less than to describe the sequence of impressions enjoyed by such an ideally possible percipient.

Now it is our contention, on the contrary, that a logic like this which grounds judgment essentially in an immediate contact of sense, has no right to explain universal propositions in terms of a hypothetical or possible experience. For if it be true that judgments are rooted in concrete perception, then assuredly each one involves an organism as receptor of stimuli and source of responses,[4] an organism which cannot be defined vaguely "in general," but must be that of a specific actual somebody. To attempt, moreover, to construe universal assertions in

[4] Cf. pp. 248, 257.

terms of the experiences of a Transcendent Organism or Perceiver that the exigencies of the theory require us to invent, involves an obvious circle. Such a device removes the basis of natural law to a transcendent world, from the obscurity of which anything may be asserted without restraint as either possible or impossible. In brief, once we assume experience *to be experienced as it is not actually experienced,* we play false to the empirical principle.

As for the hypothetical judgment in general, there are, if I am not mistaken, two ways of viewing it. On the one hand, it may be taken in a purely formal sense as expressing strict logical conditionality, in which case it is indistinguishable from ground and consequent—"If *A,* then *B,*" is held to mean, "Because *A,* therefore *B.*" But, although the judgment is here called "hypothetical" to signify its derivation of a necessary conclusion from an assumed postulate, it has really at bottom no proper function or distinction of type differentiating it from abbreviated syllogistic inference. In the true sense, it is not a judgment but an inference. On the other hand, however, the hypothetical

judgment may be taken in a material sense in any given case as expressing a merely empirical dependence or correlation of phenomena. "If it rains, we shall carry umbrellas." Here, the judgment is ordinarily meant to signify a sequence of activity phenomenally contingent upon certain initial experiences. But, although the hypothetical judgment gains distinctness as a type when construed as a declaration of empirical contingency, this meaning is plainly incompatible with the assertion of necessary connection between phenomena. Unfortunately, this fact is not always kept in mind. And there appears constant danger of insinuating a claim to formal necessary connection into assertions of purely material sequence. Indeed, as it seems to me, there is a tendency in the whole view that interprets universal judgments hypothetically to transform the formal bond of ground and consequent between terms in the hypothetical judgment into a specific material connection of the content: the object being to confer thereby upon certain laws of science a binding force of authority beyond the possibility of attainment from mere experience.

IV: Can Hypothetical Judgments Express Necessary Connections between the Data of their Content?

Presumably, there are many who believe that the hypothetical judgment can express a logically necessary connection between the concrete data of its antecedent and consequent; and hence, that any scientific law so regarded may be understood as the statement of a specific apodictic relationship between phenomena, which holds good, moreover, apart from any assumption as to the existence of the phenomena. In this last, we have a feature of the hypothetical judgment which especially recommends it to these advocates. This type of judgment, they maintain, is exclusively concerned with affirming necessary connection, and posits nothing as to existence. Thus, when we formulate the law of inertia by saying that "If a body is free from outside force, it will continue in a state of rest or of uniform motion in a straight line," what we really assert is a pure connection apart from all considerations of actuality. This is shown, they say, by the fact that the law is generally accepted despite the absence of a sin-

gle unequivocal instance in our experience of a body in the condition prescribed by the law (viz., uninfluenced by some force of gravitation, friction, or other). Indeed, we should hold it to be true even though every particle of matter in the universe were annihilated; just as we should maintain the truth of the statement "If you drink arsenic, you will die," even though there were no drinkers of arsenic. The advantage *par excellence* of the hypothetical judgment is then, from this point of view, that it enables us to make true assertions regarding ideal and contrary-to-fact states of affairs without implying that they exist. And because it alone appears able to assert apodictic relationships free of existential import, the hypothetical judgment is held to be fundamental to, and irreducible to, the categorical type.

So paradoxical a position deserves to be closely examined. In the first place, as it seems to us, a mistake is involved in the attempt to construe the hypothetical judgment interpreted in this way as fundamental. To us it seems fairly evident that the categorical type represents the more elementary form or main root

from which the other judgments have sprung. For not only are the hypothetical and disjunctive propositions essentially compound, being composed of two or more categorical judgments combined together;[5] but, in addition, only the categorical judgment among these forms makes a direct, unqualified assertion about reality, and, as such, alone has the property of material truth or falsity attaching to it. In the second place, it seems highly questionable how, on this view, there could be laws of nature that would hold even though there were no nature. For just as nature is conditioned for us by our perceptive and retentive faculties, so natural laws also would appear conditioned by these same faculties. In what conceivable form could the law of inertia hold if no matter existed? Or in what sense could arsenic be a mortal drug if there were no living beings to take it? Clearly, in

[5] In contrast to the view of Bradley and others that all universal judgments and scientific laws are hypothetical, Bosanquet seems to hold that the disjunctive judgment is both fundamental and superior to other forms of judgment because of its power to express classifications. (Bradley, F. H., *Principles of Logic,* 2nd ed., bk. I, ch. II, sects., 44-46. Bosanquet, B., *Logic* 2nd ed., vol. I, ch. VIII, especially p. 328; II, pp. 149-198.)

such a case, the antecedent of the judgment would never be formulated, much less the consequent deduced.

And if the believer in the apodictic force of the laws of nature retreats once more upon a supposititious or hypothetical experience, he will be no better off. Suppose he tries to divest the warning against arsenic of all existential import by stating it in this fashion: "If certain unrealized conditions were realized (viz., if somebody took arsenic) then somebody would have certain experiences he has not had (viz., death)." To this, we, as critics, may retort that the import of such an assertion is no more than that "If things weren't as they are, they would be as they are not." Either the meaning is a worthless tautology (since no information is given as to the unknown, contrary-to-fact conditions of such a world), or it is simply, as has been said, a veiled attempt to capitalize ignorance as knowledge and to foster obscurantism. "If artichokes were butterflies, giraffes were mustard-seed"; "If the moon were green cheese, we should expect a strong odor on moonlight nights"; "If a guinea pig is picked up by his

tail, his teeth will drop out." In such propositions we merely show how the content of the antecedent, if taken as without existential import yet endowed with apodictic force, can be made to imply anything.

Our conclusion is that, where the "given" is accepted as the core of certainty in a theory of knowledge, it is impossible to rescind all connection with concrete fact. In such a case, reality, as it is actually experienced, must furnish the basis of interpreting nature. To attempt to shift the center of gravity to another world, to a connection between data that do not necessarily exist, is to remove all firm supports from knowledge. In what sense could the judgment "Oranges are yellow" be so if there were no oranges or perceivers of them? Plainly, in no cogent or coherent sense so long as judgment is grounded in sensuous contact; for it seems impossible that judgments should spring from actual experience, describe connections that apply to it, yet possess their validity in total independence of it. In other words, if judgment is conditioned by concrete perception, it cannot be a matter of indifference whether the state of

affairs affirmed or denied therein is, or is not, a matter of actual perception. The effort after necessary connections defeats itself when it is imagined that the laws of phenomena can be defined without contradiction both as apodictic forces uniting phenomena and as valid irrespective of the existence of these phenomena.

Surely, since Hume, everyone with a drop of empiricism in his veins has been convinced of the impossibility of attaching any specific logical force or necessity to the content of phenomena. Because scientific laws are essentially the record of a finite number of empirical observations and generalizations from the behavior of actually existent events, their formulation can never be freed from material dependence and contingency, but remains always in a measure problematic. Although in certain fields (like mechanics, for instance) a high degree of success appears obtainable through separating certain formal quantitative relations from the complex details of the particular material, time, and place—so that, granting the premises, the conclusions seem to follow with mathematical necessity—even here there always remains a ques-

tion as to how closely these laws of our construction fit the literal facts of the physical world. In other words, we cannot be sure that the empirical sequence of one particular state of affairs upon another exactly coincides with the formal conditions of the mathematical law that we set up. And though the hypothetical judgment is generally held to be capable of expressing either of these modes of connection, serious errors arise when an observed correlation of the first type is mistaken for the universal necessity of the second.

Sometimes special tests are introduced as aids in distinguishing whether a given hypothetical judgment expresses a genuine necessary connection in its content, or a mere empirical sequence. Thus, if the proposition "If *A* is *B*, then *C* is *D*" affirms a real apodictic linkage, it is said, it should be possible to translate it without loss into its logical equivalent "Either *A* is not *B*, or *C* is *D*." And where it cannot be so translated, we may conclude the connection is not apodictic. For instance, the statement "If this is a triangle, it has not four sides," can be readily converted into its disjunctive equiva-

lent; on the other hand, such assertions as "If wishes were horses, beggars might ride," or the law of gravitation, clearly reveal their contingency and lack of inherent necessity when retranslated in the form "Either wishes are not horses, or beggars would ride" or "Either a thing is not a body, or it attracts all other bodies with a force directly proportional to the product of the masses and inversely to the square of the distance."

Another less decisive test offered is that of pragmatic verifiability. A hypothetical judgment is, from one point of view, held to express the content of a "possible experience." Thus, when Copernicus made his revolutionary assumption of standing on the sun, we may imagine (it is said) that his thought took the form: "*If* I were standing on the sun, the planets (including the earth) would appear to be revolving round me." Now, although his antecedent described nothing actual but only the supposition of a possible or conceivable experience, the consequent (presuming the law were true) might be supposed to exhibit certain secondary effects concretely verifiable by observation on

the earth. In other words, a law of nature, even though stated in terms of merely possible experience, must, if it is really true, have some discoverable connections with present, actual, human experience. Accordingly, it is said, the critics of Copernican astronomy were justified in demanding "If the hypothesis is true, what sensible consequences follow from it?"

If none had been forthcoming, presumably the theory would in time have been declared false. But since, as things turned out, an increasing series of apparent consequents were established as experimental facts, the antecedent hypothesis was held to be regressively authenticated. Only, it is claimed, by some such pragmatic method of justifying a theory by its fertility in empirical results can we even partially test the abstract bases of science. Laws like Newton's law of inertia or Einstein's principle of equivalence, for instance, are expressions of scientific imagination that extend far beyond the range and criteria of ordinary experience. Presumably they have to be tested therefore by means of certain "ideal experiments"; that is, by the supposition of certain

imaginary states of affairs and the subsequent discovery of concretely observable consequences that ensue from them.

But, obviously, this sort of verification-process is never finally completed. Nor is there any ultimate assurance that the fruitful results that seem so firmly attached to a given hypothesis to-day may not have to be construed as the consequent of a different hypothesis, which fits the facts a trifle better, to-morrow. Without any wish on our part to deny the practical convenience of testing hypotheses through their concrete consequences (a procedure which has been strongly emphasized by pragmatic logic), it must nevertheless be duly noted that such a method cannot pretend to formal certitude. The very fact that it involves the fallacy of "affirming the consequent," however practically unimportant, at least robs the method of any power to disclose necessary connections.

We may conclude, then, that the dependence of hypothetical judgments, as ordinarily interpreted, upon the data of experience for their significance, removes them from the realm of certitude. Although the consequent depends

upon the antecedent, to this latter in turn is assigned an ungrounded dependence upon the existence of the facts stated in it. Yet, in the nature of the case, the existence or non-existence of these facts can hardly ever be determined with certainty. Unlike formal truths, material states of affairs elude unconditional determination and remain almost always in a borderline realm of probability where a measure of contingency cannot be wholly discounted. Because the hypothetical judgment seems thus contingent upon conditions in part unknown and never beyond reach of doubt, its content can rarely, if ever, be known to express necessary connection.

It is only an error to suppose that a mere exculpatory washing of hands by adopting the initial word "if" can rid the judgment of the problem of existence, or that the mere ungrounded *claim* to necessary connection is sufficient to achieve such a linkage of the parts. The illusion which sees in the hypothetical judgment a chance to enjoy the advantages of participation in the worlds both of existence and of pure logic, while at the same time escaping the responsibili-

ties of both by avoiding any commitment to these worlds, must go the way of all double dealing. For no such convenient form can be devised by which reflection can be swung between two worlds, at once *of*, yet *not of*, either. Because of these things, the claims of the hypothetical judgment must be dismissed as offering no genuine basis for valid reasoning.

CHAPTER IV

THE INFERENCE AS THE FUNDAMENTAL UNIT

I: The Inference as a Logical Process and as a Psychical Act

HAVING disposed of the various errors growing out of the belief in the primacy of the judgment, we may return to a consideration of the claims of the inference as the basic type of understanding. Unfortunately, inference is a term that suffers in general usage from not a little ambiguity of meaning. On one side, it is used to designate a *passage of thought* from a judgment or judgments utilized as grounds to some new judgment. On the other, perhaps more rarely, inference is taken to stand for a purely formal *logical operation* independent of psychical activity as a natural event, and entirely sustained through the articulation of its structure. Between these two usages, there is a certain undeniable incompatibility; for, wherever the passage from ground to conclusion depends upon the mental activity of an existing subject, the psychological, physiological, and

physical forces of the environment have to be taken into account as, to a greater or less extent, guiding and controlling the knowing process. In so far as the inference is regarded as really proving anything, however, its advance must be assumed as due, on the contrary, to the purely necessary pattern of the reasoning itself. In this sense, inference may be studied without reference to thinking as a natural process wherever, in Aristotle's phrase, "certain things being posited, something else than what is posited necessarily follows merely from them." It is with inference in this latter significance as a strictly formal process that we are here concerned.

Yet, in maintaining the independence of inference as a logical process from inferring as a psychical act, it is to be noted that we do not claim this as a discovery resulting from the mere inspection of consciousness. Unlike certain realists, we do not believe that a simple analysis of distinctions within the meaning or intention of knowing can of itself establish the independence of the object of knowledge. Yet, although the mere claim of the inferring act to

mean or *refer to* an intellectual relationship subsisting irrespective of it can never, in our opinion, prove the reality of inference as external to, and unaffected by, consciousness; nevertheless, it seems as if the reflective comprehension of inference on its own terms could establish its independence in another way as signifying the self-dependence of the process or the maintenance of its operational integrity without appeal to anything other than itself.

Even at the risk of a digression, the point here seems worth developing. Both realists and idealists, as it appears to us, have succumbed to a common error in regard to what is known as the egocentric predicament. Both have argued from the same premises (namely, our impotence to separate the object of thought from the thinking act), but they have argued to opposite conclusions: namely, on the one hand, that the object is dependent on the thinking activity and, on the other, that it is independent of it. But the mere inability of the inferring mind to escape from the circle of its own mental processes —the bare fact that I can no more outrun my thoughts than my shadow, although constantly

striving to do so—proves nothing as to the relation of the object of knowledge to the knowing activity. After all, the fact that the reflective activity is always present (even when declared absent), in conjunction with the object of reflection, imports no more than a structural peculiarity of our psycho-physical make-up. Any real evidence to show that knowledge can successfully transcend the limitations of subjective consciousness and lay hold of forms and processes independent of psychical activity, would have to be of a very different sort. Such evidence would have to show that there are certain objective wholes and configurations that are not capable of being modified to suit the subjective preferences and psychological peculiarities of the human mind, but that possess a coerciveness of their own which enables them to impose on consciousness their own terms and conditions.

And, as a matter of fact, it would appear that there are such wholes discoverable by thought in the case of certain systems of relations. These structures hold for different individuals, places, and times, yet cannot be ar-

bitrarily altered to fit the needs of the mind to which they occur. Moreover, they exhibit their independence not simply by imposing themselves upon the mind (regardless of its wishes) as brute facts, but by convincing the mind on its own terms. For certain logical configurations have the remarkable property that their very comprehension entails acknowledgment of their validity; so that, in the end, their integrity is vouched for both by reason and by experience, both by themselves and by the minds that know them.

Postulational systems like Euclidean geometry furnish the best examples of such wholes, although any deductive inference is an illustration of the essentials of the process. What is strangest about such systems is that their terms and postulates are apparently chosen by a series of free subjective acts; yet, once chosen, they acquire an independence and logical momentum of their own. That is, once laid down, these rules must be conformed to not only by the operations of the system but by any minds that would understand its workings, since only so can the system be tested for truth or falsity on

its own terms.[1] Indeed, so fundamental is this requirement that systems should fulfil their own conditions and conform to their premises in consistent fashion, that any attempt to conceive of its invalidity only confirms its validity.

Sometimes this state of affairs (this logical predicament, as it were) is mistakenly identified with the egocentric predicament; that is, thinkers maintain that the paradox of reaffirmation in denial, which holds of the logical relations involved in system, is nothing but a psychological limitation of our factual make-up. Yet, plainly, it is something very different to discover that the knowing activity is always present, even when declared absent, along with the object of knowledge, and to learn that the logical principles of consistency and coherent system (as constituting the structure of the object and the criteria of knowing) are recognized as formally valid even when declared psychologically invalid. This latter, since it lays bare the supremacy of form over content, enables us to transcend psychic existence in knowing and to lay hold of the formal pattern of the

[1] Cf. pp. 194-199.

objective world. That whatever can have truth or reality predicated of it must conform to the requirements of systematic consistency is a rule that must be accepted not simply because these logical conditions are found to be omnipresent facts of experience, but because the very distinction between truth and falsity resides in them, so that they are conceived as true even when declared false.

II: The Nature of Inference

Such reflections as the foregoing are not without bearing upon the question of the status of inference in reasoning. As has been pointed out, whatever has logical priority must be something that, though other things depend upon it, does not itself depend upon them. But, among the various factors of intellection, inference alone, as it appears, can have this sort of independence justifiably claimed for it; for of inference alone can it be said that it has its essence strictly in its form. Whereas the discrete judgment or proposition depends for sanction of its bare unsupported statement upon an alleged correspondence between its meaning and the nature

of external facts, the truth or falsity of the inference *qua* inference, on the other hand, depends ultimately upon whether its parts fit together in a way that is rationally inevitable. Thus, you may infer, if you like, that, since Caesar, Pompey, and Napoleon were men, and men are pigs, Caesar, Pompey, et al. were pigs. In other words, you may reason about any objects of any kind, and postulate states of affairs absurdly contrary to facts as they appear; and, none the less, provided your structure has a certain integrated form, it is sound as inference. Thus, the inference, as opposed to the judgment, is self-determining in that the validity of the relationship cannot be destroyed by the mere incorporation in it of meanings devoid of correspondence to actuality. For, in those cases where the single discrete judgment appears to enjoy a formal validity irrespective of its content, it is always (as we tried to show) a cryptic inference; that is, it is either an immediate inference or a mediate one compressed in expression, in which the relation of subject and predicate is strictly one of ground and consequent: "Because *S*, therefore *P*." On this and other

accounts, the inference may be asserted as constituting, in our opinion, the simplest complete unit of reasoning. A character of abstract reflection that students of the subject are never tired of pointing out is that it enables us to arrive at conclusions that hold true of *any* thing or *some* things, and not merely of certain particular things. Leaving questions regarding the specific character of the data and the verification of matters of fact to science and history, abstract reasoning as such concerns itself rather with the discovery of certain underlying operations or systems that hold good independently of the particular content of the systems.

When we come to examine the nature of inference more closely, we find it revealed in a variety of forms. Although no claim is made as to the exhaustiveness of the list that follows, presumably the more prominent types are included in it. To begin with, mention should be made of immediate inference which, however its various formulations are construed, is usually recognized as rooted in the same principles as the syllogistic and functional deductions to be mentioned hereafter. Secondly, in the chief

form of the syllogism itself, a new whole is obtained through the combination of a universal proposition and a particular instance. Thirdly, there is a large class of functional and mathematical deductions in which notions of magnitude and serial order play a great part, and which, at first sight at any rate, seem not to be affiliated with the syllogism. Examples chosen at random to illustrate this kind of reflection are, for instance: composition by addition $(x < y)\ (z < w) < (x + z < y + w)$;[2] equations involving variables, such as $x^2 = 2yz$, in which certain things are true whatever values are inserted in x etc.; and inferences involving time as a serial order, as "Since x is prior to y, and y is prior to z, then x is prior to z." Lastly, induction is generally regarded as a distinct type of inference, by means of which we pass from *some* to *all*, obtaining on the strength of a limited number of experienced data a conclusion applicable to an indefinite collection of largely unexperienced data.

In attempting to formulate the general char-

[2] I.e., Granted that every x is y, and every z is w, then everything that is either x or z is y or w.

acter of inference common to these various types, what stands out first is that they all involve a process in which a datum is taken as the condition from which a certain result follows as a consequence. In the case of a series, for instance, the antecedent member or members must operate as the adequate ground of their successors by the law of the process; while, even in immediate inference, the orientation of a statement in the context of its fundamental logical relations may reveal results that are inevitable, although they were by no means seen to be present as obviously included truisms at the start. In reflection, the product is always something distinct from the elements that combine to give rise to it. We cannot say of inference, as of meat in a sausage grinder, that what goes in must come out—that and only that—albeit finely mixed. On the contrary, something synthetic is gained in the process, something suggestive of cumulation, progression, of independent wholeness, and not of dead iteration. And this novel combination of terms in the resultant is achieved, it would appear, through some impetus of logical identity by which the different

elements are assimilated to a common basis of reference and thence, by means of it, related to each other.

III: Identity in Inference

Something has already been said as to the nature of this formal identity so essential to the generative force of inference. That it cannot be simply of an analytic kind seems evident, since the self-identity of a term with itself can scarcely link one thing with another. Were identity purely of this negative, reduplicative sort, presumably only a shadow of reasoning would remain in processes like conjunction and subsumption. Thought would be like threading beads on a string or putting apples in a basket; in other words, nothing but a simple operation of putting items along side each other or into a containing class. Fortunately, however, there is, in addition to this type of identity in thought, a synthetic identity of a formal kind which serves as the driving impetus of the process in achieving fresh results, and in forging new linkages and connections. By means of it, reflection appears able to derive conclusions

that are not to be found in the premises taken either separately, or conjunctively, or merely subsumptively, with respect to each other. Historical and scientific research relies constantly on such deductions as where, for instance, a biblical scholar concludes that John the Baptist was a priest on the basis of the internal evidence that he was the son of a priest and that the sons of priests followed their fathers' profession; or a chemist, knowing that the volume and pressure of a gas vary inversely (Boyle's law), concludes that, since he measures 200 c.c. of a certain gas at atmospheric pressure and the barometer reads 742 mm., at 760 mm. barometric pressure, the volume of the gas should be compressed to $(742/760) \times 200$ or 195.3 c.c. What interests the logician here is not so much the new matter gained, if any, but the fact that a fresh whole or combination of terms is apparently arrived at through the presence in the premises of some common denominator or integrative identity.

Yet this identity of the middle terms, or this sameness introduced by the law of a series which renders the members expressible as functions

of each other, would not seem to be of a qualitative kind. If two peas in a pod are said to be absolutely alike, their identity with each other still holds in a very different sense from that in which each is identical with itself. Usually, no doubt, all that is meant is that it is hard not to mistake the one for the other. Yet it is not likely that their absolute likeness could be upheld qualitatively and in detail; since, if carefully examined, they would almost certainly exhibit minute differences in size, shape, and texture. As a matter of fact, the sense in which terms and objects are taken to be identical by the understanding for purposes of comparison is quite independent of, and compatible with, the preservation of their qualitative uniqueness and individuality.

There are, however, some who hesitate to admit the validity of this notion of a positive, formal identity beyond qualitative differences. Such identity, they would say, amounts to no more or less than a psycho-physical inability to discriminate dissimilars; and is of the same sort as when, in comparing two musical notes with a third lying between them, I take them to be

identical with each other because I am unable to distinguish either from the middle note, until brought to a realization of error by striking the first two notes together and perceiving their difference. Now identity in inference may be said, following this line of criticism, to turn on a precisely analogous failure to discriminate the unlikenesses between the so-called middle terms. If, for instance, I reason that "whatever gravitates is material, and since the light-rays from a certain star appear subject to gravitation, these light-rays must be material," some critics may say that the only identity here in the middle term rests in the purely negative and psychological failure to detect the difference between the theoretical formula for gravitation and the rough empirical approximation in the particular case. But, once identification is construed as synonymous with mere absence of discrimination, the question arises how it advances knowledge. For failures in differentiation, after the discrimination has been made, are generally regarded as illusions and errors—the very opposite of knowledge. Thus, when analysis has separated the two musical notes, their confu-

sion must ever afterwards be considered a mistake. According to the above psychological interpretation of identity, therefore, progress in knowledge should come about through the elimination of likenesses and the discovery of differences, instead of through the detection of identities and assimilation to a common denominator. Yet, in the actual advancement of knowing, the very opposite would seem to be the case.

The true identification of terms in inference is apparently formal and implies not literal coincidence or superposition, but only, in our opinion, that one term can be treated as equivalent to another. Thus x is identical with y if whatever you can say about x you can also say about y, and *vice versa,* in respect of their form. In the syllogism, for instance, this capacity of one middle term to stand for another may be unaffected by difference of distribution, or by distinctions of intension and extension (within limits) in the interpretation of the terms. In the first figure, for example, there is clearly no identity in the sense of literal correspondence between the middle terms. For, while the major premise expresses a law of universal functional depend-

ency, the minor refers to some concrete particular; and obviously the identification between an ideal formula and an empirical instance cannot be one of bare superposition void of all difference. Very frequently also the middle term will be distributed in one premise but not in the other; or most readily construed in intension in one and extension in the other. Indeed, the classic syllogism about Socrates is said by some to escape the charge of *petitio* only through the interpretation of the middle term (man) in intension in the major, and in extension in the minor where Socrates exemplifies the class. But, as it appears to us, the whole question whether terms are to be taken in intension or extension, or as a patchwork of both, is irrelevant to the matter of identity; the distinction between the two interpretations being, in our opinion, merely one of the kind of terms involved (whether abstract or concrete), and having nothing to do with the structure of inference as such.[3] Once the nature of inference

[3] Jevons, W. S., *Principles of Science,* 3rd ed., bk. I, ch. VI, p. 119. "We state the same truth when we say that 'mortality belongs to all men,' as when we assert that 'all men are

is grasped as a purely formal process independent of existence, it becomes a mere matter of convenience whether the content is held to express the *relationships of conceivable classes* to each other or the *linkage of attributes.* Any charge that an inference may be true in intension though false in extension, or *vice versa,* must therefore, as it seems to us, be repudiated by a strictly formal logic.[4]

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mortals.' Here we do not pass from class, but from one kind of term, the abstract, to another kind, the concrete."

[4] A number of seeming exceptions have, of course, to be explained. Take the following example:

No bootleggers have crossed the border.

All bootleggers will be arrested for violation of the 18th amendment.

Hence, some to be arrested for violation of the 18th amendment have not crossed the border.

Now, many logicians would hold (in contradistinction to the position maintained above) that this reasoning makes sense only so long as it is taken in intension as an interconnection of meanings; and that it becomes false as inference as soon as it is translated into extension. For since the subject of the conclusion (those arrested for bootlegging) remains a class with no existent members (so far as the inference informs us), it seems impossible to make any statements having existential import regarding them as things that do not exist.

To this, however, it may be replied that true inferences can be drawn concerning classes taken in extension whose members do not exist. All that is necessary is that membership

What is really important is not the kind of terms involved, but the necessary connections between them. Yet, in order to express these connections, the elements of inference must be of a character to lend themselves to different interpretations, and not to one only. For this reason, the terms of the inference must be fundamentally regarded as universals, and not as mere particulars. Otherwise it is hard to see how thought can advance beyond the given; since, if the elements of reflections are taken as atomic particulars, they would seem to have no bearing beyond themselves and to afford no foothold for interconnections. Where a term is strictly particularized, its form appears indistinguishable from its matter, thus rendering it inapplicable to other values and substitutions. Observation of thinking will also show how it

in these classes should be conceivable and not self-contradictory; no correspondence with the actual facts of existence is required. From this point of view, the only invalid inferences of this sort would be due to the inconceivability or contradictoriness of the terms themselves, as if, for instance, one should say that "no squares of whole numbers between 9 and 16 are unextended bodies; all squares of whole numbers between 9 and 16 are square circles; hence some square circles are not unextended bodies."

everywhere moves through the universal, taking its object not as merely blunt, uninterpreted fact but as a sign or typical instance representing a class, from which it is possible to move on to another class. Thus, even the individual, Socrates, is never wholly *sui generis,* but (if he is to be thought of at all) must play his part as, for instance, the father of philosophy, an Athenian moralist, or the logician's stock specimen of the *genus homo.* Because the elements of reflection seem to embrace both a variable and a constant significance—being conceived, on the one hand, to stand for any one of a number of specific values while, on the other, they possess some definite, particular meaning which includes a whole class of values—because of this, any term of inference may be construed as a latent or unspecified universal.

IV: The Form and Matter of Inference

An attitude which is frequently associated with the treatment of terms as fundamentally particular is that which claims that the form of inference depends upon its matter or content. To this, we may devote a few words of criticism.

According to a rather widely accepted view, the validity of an inference is conditional upon the fulfilment of two requirements: first of all, the premises must possess "material truth," that is, they must be correct as statements of fact; secondly, assuming the fulfilment of this first proviso, the conclusion must follow from the premises with logical necessity. However, these stipulations in advance, as it appears to us, are such as to render valid inference practically impossible. For how can valid deductions ever be obtained if the establishment of matters of fact be made their prerequisite? Not only is the content of our premises in daily life seldom or never certain, but even in science and exact thinking the achievement of precisely accurate statements in regard to empirical events remains an ideal to which the scientist only provisionally and tentatively approximates. But, if it once be admitted that certitude is practically unobtainable regarding matters of fact, there can be no opportunity to enforce the further requirement; namely, that the conclusion should follow logically from the premises. For if the first condition cannot be met, obviously

the second, which depends on the first, must remain unfulfilled. We may conclude, therefore, that if the form of inference were really conditioned by its matter, indeterminate probabilism and speculative conjecture would be all that would be ultimately gained under the name of knowledge. Since, in addition to the previous difficulties, we should also be obliged to wait until all potential data were in and the train of events at an end before being able to make even an approximately correct pronouncement regarding the *matter* of knowledge, the result would presumably be indefinite postponement of the possibility of discovering any valid *forms* of it.

Yet even skeptics have usually to admit that there appear to be certain general operations or structures of intellection which remain the same, and hold equally well, in connection with different subject-matters. One and the same general process is traceable in induction, for instance, whether the phenomena be gases, birth-rates, the path of comets, or the growth of cells. To be sure, you must have analyzed the facts and have them pretty well in hand before your conclu-

sion becomes of any value as induction, and, in this sense, the form of the process may seem to emerge from the matter. Yet it is only on the supposition of the validity of the procedure itself, used similarly in different connections, that the method gains significance. In the last analysis, those logicians would seem to be right who construe induction, furthermore, as akin to deduction. For the first need in inductive reasoning is always for some general idea or tentative guiding principle in the light of which the manifold facts may be organized. Moreover, to obtain such a provisional major premise no blind "leap of faith" would seem to be necessary, but only special emphasis upon and recognition of the non-existential, universal aspects of the particular subject-matter. Since everything, as we have urged, enjoys the ambivalent nature of being both a "this" or an "any," thought is entitled to choose in a given situation whether it will take its object as a particular or a universal. For instance, in confronting the state of affairs represented by a table, one is equally justified in asserting "This is a table" or "Matter in the form of a rectangular slab

with four legs exists"; just as in confronting the spectacle of a solar eclipse one has the choice of declaring "This is an intervention of the moon between the sun and earth" or "Intervention of the moon between the sun and earth is an observable fact." If choice is made of the latter proposition in either case, one has a ready-made universal proposition on one's hands. Once having acquired a general notion, which for the time being you accept, the next step is to bring it into connection with special cases or concrete data. Accordingly, a varied selection of instances may be brought forward for subsumption under the general idea. If they fit in with the major premise without exception, and if they constitute a sufficiently representative range of observations, the law may be taken as more or less established. But if, on the contrary, some of them are found to contradict the tentative major premise, the latter will have to be discarded. In that case, it becomes necessary to look for another guiding idea to be tried in turn through its consistency or inconsistency with concrete instances. Thus, induction is seen to work by a process of elimi-

nation as it were, whereby various provisional major premises or alternative hypotheses are tested for their coherence with facts. That law finally remains in possession of the field and is adjudged trustworthy which appears, on the one hand, uncontradicted by experience, and, on the other, enjoys a body of examples congruous with it.

In the end, it would seem that any view which attempts to reduce the form of thought to ultimate dependence upon its matter is forced into an untenable position. For, once it is recognized that certitude is never obtainable in the realm of facts, it follows from this and the previous stipulation, that reasoning can deal only with indefinite probabilities. The uncertainty of the premises must inevitably infect the whole process. To say that "All propositions are therefore only probable" is apparently to affirm that, although all propositions are either true or false, I can never have absolute assurance in any given case that an assertion is one or the other. But probabilism in this sense harbors contradiction. For, in assuming a basic disjunction of alternatives into true and false, I am presupposing

certain formal knowledge as fundamental to my ignorance. Thus, when I say "The proposition which I now utter may or may not be true," it is clear that the certitude of the operation of the formal law of excluded middle is cut across by my declaration of the uncertainty of material knowledge with a resultant paradox. And if my formal knowledge is here dependent upon my material ignorance, even the reality of the disjunction as a meaningful utterance is doubtful. Moreover, it is contradictory to suppose that a proposition may be either true or false, if at the same time we suppose that it *is true* and not false (this latter being an unavoidable postulate, in that every proposition assumes its own truth). Observing, therefore, the predicament resulting from the supposition of the supremacy of the content, our conclusion is that the very assertion of probabilism as an intelligible hypothesis can only be achieved by accepting the contrary notion of the sovereignty of form over matter.

V: Inference and the Paradoxes

Mention of this paradox of probabilism recalls certain difficulties charged against formal inference which seem to threaten its validity from a new quarter. These difficulties, to which attention has been directed in recent years, especially by Bertrand Russell, arise in connection with a group of so-called *insolubilia* or paradoxes involved in reasoning of which "the liar" still serves as the classic type. The precise trouble is this. The doctrine of formal inference maintains that the structure of a valid inference is valid for all values that may be substituted in its terms; whereas, as a matter of fact, it would appear that its validity is destroyed when certain contradictory values are introduced. Yet, if this be so, and if the forms of inference fail to hold for all subject-matters, there is good ground for believing, after all, that the validity of the form is contingent upon the nature of the content.

In taking up the question, it is first desirable to make clear what constitutes a paradox, and then to discern which among the so-called paradoxes, if any, place reason in a difficulty. One

of the pitfalls to be avoided is that of taking the concept too broadly, since, in that case, its essence may escape us. Although the paradoxes are generally indicated by single categorical propositions (such as "I lie," "All generalizations are false," "There are no laws without exception," etc., except where they are stated symbolically), they furnish excellent illustration of the fact that a proposition has import only through virtually including or being included in an inference, since it is in these attendant inferences that the paradoxes really consist.

Let us begin by asking whether a paradox may be sufficiently defined as a proposition which applies to itself. Does a proposition which has reference to itself necessarily involve a contradiction? Such propositions, it is found, fall into two groups: those that are wholly self-referent, and those that have an extra-reference in addition. An example of the first sort is the statement, "This proposition is true"; and one of the second is discoverable in the law of excluded middle expressed in the form, "All propositions are either true or false." Now there is no denying that, when a proposition is

included as a value in one of its own terms (as in the foregoing instances), some peculiar consequences follow. In the first place, such assertions give rise to infinite series. Thus, when I say "This proposition is true," the whole statement is understood to stand in apposition to the subject "this proposition." A better expression, then, would be "This proposition, 'This proposition is true,' is true." Yet, no sooner is this fuller form stated than it, in turn, will have to be made the subject of a new expression. In this way an infinite process will be generated, due to the fact that the original proposition contains itself as a value of one of its parts. The same thing holds likewise of the law of excluded middle as cited above. When I aver that "All propositions are either true or false," I plainly mean to include this very statement within the class of "all propositions." Hence my remark, if developed, becomes "All propositions, including this one that 'All propositions are either true or false,' are either true or false," and so on *ad indefinitum.*

Before pausing to consider whether the generation of an endless series is enough to consti-

tute a paradox, let us mention two other characters of this class of universal assertions which include themselves as values in their own terms. One of these further characters, which is closely allied to the former, is known as that of illegitimate totality. Whereas a proposition about "all propositions," like that above, claims to cover a set which has a total, the fact is said to be overlooked that it contains members which presuppose this totality, and therefore that it cannot have a total. In this sense, any statement regarding the class of "all propositions" fails to envisage totality. Precisely the same difficulty is mentioned in a later chapter in discussing the possibility of making assertions about the "universe." The question is raised whether it is ever really feasible to refer to an all-inclusive subject-matter, for the reason that the very notion of wholeness itself constitutes a further item not included in the whole referred to. Reflecting upon this charge, some persons come to regard the concept of totality as illegitimate. Lastly, the imputation of vicious circularity is now and then urged against self-referent assertions. A circle is said to re-

side in the assumption that a proposition (or class) can be a member of itself, and thus be self-substantiating. Where whole and part are virtually the same, there would seem to be a *petitio* in any attempt to establish the one by the other. For instance, our belief in the universal principle that "All propositions are either true or false" seems to rest upon our acceptance of the right of this (or any) particular proposition to assume its own truth (and to exclude its falsity)—a belief which we, in turn, justify to ourselves through our assent to the exceptionless operation of excluded middle.

But the foregoing charges, that propositions which apply to themselves involve infinite series, illegitimate totality, and circularity, seem to us hardly enough to constitute them paradoxes. In fact, any definition of a paradox appears to be too wide if it includes basic propositions like the law of excluded middle—such propositions as apparently demand acceptance as sound if anything is to have meaning. Indeed, were all self-referent propositions paradoxes and freighted with the fallacies above mentioned, we should find ourselves practically un-

able to utter a single universal statement regarding inference, truth, logic, thought, or expression, without contradicting ourselves within the meaning of our terms. No logician could say, for instance, "All propositions that refer to themselves are paradoxical" or "For all values of ϕ, ϕ (x) implies ϕ(y)" without being convicted of paradox out of his own mouth (since "all" here involves self-reference). Hence, since upon this view the very definition of a paradox implies a paradox, the position would appear to be self-refuting. Moreover, as was pointed out earlier, the basic prerogative of reasoning everywhere assumed is its capacity for self-transcendence and the power to make statements about a totality while remaining within it. In fact, in all its relations, intellect is obliged (in so far as it escapes the circle of inner consciousness) to assume its capacity as a theoretic *means* to reveal an *end* or object of which it is a part, without necessarily contradicting or falsifying the object. Once deprive reflection of this power to grasp a whole while continuing as a member within it, and its basic function would seem to be destroyed. On

the other hand, if once the validity of self-transcendence be admitted, the charges of circularity, illegitimate totality, and infinite seriality lose their paradoxical character. Thus, once it be granted that reflection enjoys both an immanent and transcendent capacity to compass a whole of which it is a part, there is no longer any vicious circularity in supposing that a proposition which refers to a class of which it is itself a member should have its respective aspects mutually support each other. Similarly, the notion of illegitimate totality, which turns upon the illusion that a proposition cannot include itself as part of its object of reference, is dissipated by the same acceptance of rational self-transcendence. As for the tendency of self-referent propositions to generate infinite series, this may be explained as merely a marked expression of the equivalence that always subsists between conceptual parts and wholes, by which the one can be set in correspondence with the other—an equivalence familiarly exemplified, without any question of paradox arising, by the fact that a part of space or time can stand for the whole of space or time.

It is, then, apparently not enough to characterize as a paradox any proposition that includes itself in its own scope. The really genuine paradoxes are rather, as it appears, to be looked for in that small group of propositions which, on the one hand, when taken to be true assert their own falsity, and, on the other hand, if taken to be false assert their own truth.[5] Thus, if a man speaks the truth in saying "I am lying," he indeed speaks falsely when he says he lies. But if, on the other hand, he speaks falsely when he says he lies, then he speaks the truth (since if an assertion of falsity is false, it must be true). But what is it that makes these propositions, as it were, doubly self-contradictory, so that the assertion of either their truth or falsity involves the contradictory opposite? Apparently the answer is, that, where a proposition with an individual subject applies to itself yet is formulated in such a way that its content directly conflicts with its formal assumptions, we have a paradox. As such, it is sharply dis-

[5] Throughout this discussion of the paradoxes I am much indebted to Mr. E. R. Guthrie's interesting monograph on *The Paradoxes of Mr. Russell.*

tinguished from the ordinary incompatibilities, like square circles, which subsist wholly within the content. In the genuine paradox, the contradiction is rather between the matter and form, as where the content expressly denies some principle (of truth or falsity, universality or consistency) which is essential to the very structure of the proposition or the formulation of inference as such. Thus, every proposition ordinarily assumes its own truth. But when one is framed in which the content flatly and completely denies its formal claim to validity, such as the above, we have what is called a paradox. For the contradiction remains even if we specifically change the assumption underlying the proposition to one of falsity; since, in that case, the form echoes the content's denial of validity and we are confronted with a double negative (which in logic is equivalent to an affirmative) or verdict of truth, despite the fact that this is denied by the proposition's explicit assertion.

It should be noted here, however, that propositions of the character "All generalizations are false" or "No law is without exception" are not paradoxes in the full and complete sense

of the above, where $(p < p')$ and $(p' < p)$, in other words, where the proposition not only implies its contradictory but its contradictory implies it. For although, if it be assumed true that "No law is without exception," this proposition must be false, since this assertion itself claims to be at least one law without exception; it does not follow, on the other hand, that assuming the same proposition to be false (which is equivalent to its being true that "Some laws are without exception") implies the truth that "No law is without exception." Nevertheless, wherever a proposition applies to itself, yet has its form and matter mutually incompatible, its validity will be found either to be incapable of being affirmed, or incapable of being denied, or both, without contradiction.

Where other logics than common general logic are employed, and symbolisms are introduced, the list of so-called paradoxes can be considerably extended. In such cases, paradoxes like the following are frequently to be met with. "A class concept may or may not be a term in its own extension. 'Class concept which is not a term in its own extension' appears to be a

class concept. But, if it is a term of its own extension, it is a class concept which is not a term of its own extension, and *vice versa.*"[6] Now, from our point of view, this is not a genuine paradox, for the reason that the whole difficulty turns upon a conflict of particular meanings arbitrarily assigned in the definition of terms rather than upon a conflict between the general logical form of the operation and its content. The trouble, as we see it, springs entirely from the contradiction in the content of the notion "class concept which is not a term in its own extension" (as absurd as if we referred to a "*non-A A*") rather than from what follows. In brief, valid reasoning cannot even make a start from such contradictory concepts as "the class of all classes that are not members of themselves" or "class concepts which cannot be included in their own extension," since to define a class as not a member of itself is to deny at the outset the fundamental canons of identity, self-reference, and self-consistency, upon which the possibility of all intelligible conception is based. For, although in conception the

[6] Russell, B., *The Principles of Mathematics,* ch. X, sect. 101.

mind enjoys a wide freedom to define its terms as it pleases, it is nevertheless bound by certain conditions of operational integrity. And, since the contradiction in the above example subsists in the definition of terms (as in the notion of "round-square") rather than between the form and content of the reasoning, we cannot regard it as a real paradox. With regard to the theory of types in general, which is offered by Russell as a solution of the paradoxes, we can only say that such efforts to eliminate contradictions through inserting further contradictions (by the arbitrary definition of terms) seem to us more or less absurd. For any device which posits that a class or proposition shall be understood never to include itself in its own scope, inevitably runs counter to a fundamental stipulation of reasoning in favor of the exceptionless inclusiveness of reference of universal terms.

In our opinion, therefore, the formal validity of a logical process cannot be impeached by the mere insertion of some particular value in its terms. At the same time, although the form remains independent of the content, the content

depends upon form. In other words, our account of the paradoxes emphasizes that while any values may serve as the contents of a proposition without destroying its form, none the less there are certain values which, when instituted, are seen to conflict with the most general logical principles, and which are thereby negatived and declared to be invalid as contents on formal grounds. Such invalidities, more especially when they take the extreme form described above involving double contradictions, are known as paradoxes. Their significance lies in the fact that they show once more the power of reasoning to act as a criterion over its subject-matter, the supremacy of form over content.

VI: The Positive Synthetic Character of Inference

But even if it be granted that inference is, as I have argued, fundamental to both the concept and the judgment, even so this will seem to many a barren result. For reputable authorities still believe for the most part, as in the time of Hume and Kant, that general logic is solely

negative and analytic in character, and hence that the mere discovery of one of its forms as basal to the others can throw no light upon the problems of the real world. According to the commonly accepted doctrine, these formal processes are solely valuable as negative tests of what the world *is not* and for the detection of fallacies in thinking. The most that can be said for them is that they are aids to consistency; but as engines of proof and discovery they are utterly deficient. To the advancement of knowledge and the organization of positive truth they are held to have contributed nothing.

Following the tradition established by Kant, modern thinkers tend to suppose that synthetic judgments *a priori* are to be found outside of formal logic and to be incapable of formal demonstration. The confidence which they deny to purely rational relations, they repose instead in the laws and methodology of the natural sciences. These they take to embody the highest authority for truth in mankind's possession. According to this position, the canons of the sciences (such as the categories of cause and effect, action and reaction) derive their au-

thority from the fact that they constitute the *indispensable presuppositions* of experience. Exactly what is meant by this contention, however, is not so easy to say. For, although these categories are held to be conditional upon experience in that they are essentially defined through application to it, they are none the less not held to be derivative from it, any more than they are derivable from the abstract logic of the understanding. Yet the mere fact, we should urge, that experience seemingly demands, without being able to justify, certain laws, can never prove the validity of these laws. If all synthetic propositions *a priori* are contingent in this sense, we can never claim that they are universal and necessary, but only that they appear to be ubiquitous and inescapable within the circle of experience.

But if the evidence for our highest knowledge comes to no more than this, if in the last analysis we find ourselves tethered to experience by conditions that cannot bear rational scrutiny, then, surely, certitude and absolute knowledge must be relinquished as goals of our thinking. For under these stipulations, the

ultimate principles of philosophy and science would be nothing but so many *de facto* formulæ which describe the apparent admixture of elements in experience, yet which (since experience might conceivably be quite different from what it is) would remain always rationally precarious and contingent to the extent that a sudden turn of the wheel of fortune might convert the substantial-seeming events of to-day into the non-existents of to-morrow. In other words, if this type of explanation be accepted, philosophy and science must relinquish hope of an intelligible and satisfactory account of things, and must approve instead one that, in the last analysis, offers no principle of determination other than the force of circumstances.

The great difficulty of pointing out synthetic truths of unimpeachable character is one reason why even thinkers who are reluctant to accept this result remain of the opinion that only relative knowledge is obtainable. Yet one wonders why this state of affairs persists, since it would seem that nothing should be easier at any time than to confute those who deny synthetic truth to formal reasoning by the citation of so simple

and ready an example as the inference itself. For wherever reflection occurs, some new product of knowledge may be said to be generated which is characteristically different from the elements that entered into its combination. And, although no particular inference can be adjudged final in the sense of having an unanalyzable, irreducible content, the form of inference (and its matter as an illustration of its form) at any rate seems unimpeachable. Because inference and the laws of thought determine the very essence of affirmation, negation, and the derivation of consequences, no affirmation, negation, or conclusion is possible except upon the supposition of their validity. Even where the particular content of the inference constitutes a denial of logical laws and their synthetic character, the form through which the inference claims the genuineness of the discovery itself reposes on the truth of these same principles.

VII: Inference and the Infinite Regress

Yet even when this is granted, another serious charge awaits inference. Because each infer-

ence is imbedded in an indefinite series of inferences that precede and succeed it, its structure is said to fall short of the substantiality required of the ultimate unit of intellection. In other words, since every ground is the next ground in reasoning, and since everything, in so far as explained, is explained by something else, the infinite series of grounds in the inferential process would seem never to be completed; and hence the inference is never established as basal. Now, admittedly, this is a criticism not to be lightly put aside. It is apparently true that no inference is fundamental in the sense that no further inferences underlie it; and that every system of explanation contains new subjects for explanation. Yet, although any particular explanation leaves much to be explained, it cannot ordinarily be said to explain nothing. At the least, it splices up a specific problem with a general law or provisional major premise, thus showing that there is more to the situation than is speciously "given." As has been said, inference moves through the universal and accounts for things by relating them to a wider context of characters. What is more,

each of these contextual universals is grounded in a wider universal. Thus, what is a law at one stage becomes at a later stage an instance under a wider law; the classic illustration being, of course, the inclusion of Galileo's law of falling bodies under Newtonian gravitation, and the subsequent inclusion of the latter in the general theory of relativity. Although there appears to be no limit to this process of inclusion and supercession of one explanation by another, nevertheless it is notable that the later accounts seem to build upon the earlier, reformulating the essence of their predecessors in broader, more systematic form. Thus, while the answer which a specific inference gives to a question may raise new questions, the important matter is that at least it does not raise the old ones over again in the same way. Although, like the infinite regress, inference is a perpetually self-generating process, it does not, like the "bad infinite" of philosophy, involve a monotonous iteration and pursuit of an endlessly escaping fugitive idea; but, on the contrary, builds upon its anterior stages with a steady, cumulative advance.

Unlike the serial regress, moreover, inference does not admit of the concept of end-terms. While the search for grounds and consequents leads reflection into continually branching and more complex forms, it does not permit it to abandon those already won after the first novelty of discovery, or to leave them standing as isolated outposts on disused tracks of thought. On the contrary, reflection is prevented from taking any term as final by the inherent necessity of its nature which brands as self-contradictory all ungrounded grounds and detached terminal members. This being the case, there seems no alternative but to conceive reflection as ultimately a self-enclosed, concatenated process in which the totality of grounds and consequents somehow mutually establish each other.

Nothing so well expresses this double tendency of reflection toward complexity on the one hand, and unification on the other, in our opinion, as the nature of inference itself. For inference, it must be remembered, can always be viewed in two ways: either as the derivation of grounds or as the deduction of consequents. Ac-

cording to the first aspect, it is impelled to the analysis and disclosure of as many explanatory conditions as possible, and hence proceeds by multiplication and disintegration; in its other aspect, however, it betrays an opposite tendency toward unity and integration through linking different antecedents together in deriving a logical successor. Since no particular terms can be ends in themselves, reflection is kept doubling on its tracks, linking up one and the same item in a variety of contexts to the limit of its capacity. Thus, the aim of reflection would seem to be to draw the maximum number of inferences from the minimum number of terms.

This ideal receives recognition in science under the name of the principle of parsimony or simplicity. Entities, it is said, must not be multiplied beyond necessity—a doctrine which plainly attaches greater importance to the threads of connection between things than to things themselves. In other words, thought should aim at a minimum outlay in the number of its terms or assumptions, while increasing the number of interconnections to the greatest extent. The desirability of utilizing the fewest

concepts in the greatest variety of ways is a maxim to which scientific thought constantly appeals. To this tendency may presumably be ascribed the search for uniformities and the belief in an underlying unity running through the most diverse phenomena. This it is that has helped to break down the chemist's belief in the fixity of the elements, the biologist's faith in the fixity of species: both of which have given place to the assumption that a very few components, if taken in a sufficient number of combinations, can give rise to a bewildering diversity of forms. To the same ideal, Mill gives expression in his definition of the laws of nature as "the fewest and simplest assumptions which, being granted, the whole existing order of nature would result."[7] Even in scholastic philosophy this idea of "least means" appears as a commentary upon the wisdom and perfection of God who, it is pointed out, is able by the simplest agencies and from the smallest number of causes to produce the greatest possible multiplicity of effects. This impulse to conceive the number of interconnections and terms in in-

[7] Mill, J. S., *A System of Logic,* bk. III, ch. IV, sect. 1.

verse ratio and, other things being equal, to prefer those theories that multiply the former and reduce the latter to the lowest point, is part of a general tendency of reflection to emphasize the discovery of relations and applications, while, at the same time, discountenancing acceptance of terms as ends in themselves. Not that we can suppose that nature actually hews to the line of simplicity, or even that this method will make our calculations psychologically easier—thereby gratifying our natural inertia—but because reasoning, through its accent upon relations, seems to indicate the ultimacy of system.

CHAPTER V

FACTS AND RELATIONS

I: The Problem of the Nature and Status of Facts

CERTAIN positive results of our inquiry begin now to emerge. Underneath the fleeting pageantry of experience is to be discerned a groundwork of logical principles, and among these a fundamental type which determines, without wholly being determined by, others. The name of this basal unit is inference. Yet, having made this discovery, it becomes necessary for us to align other conceptions with it and to assess its value in the light of other problems. To begin with, it is understood that we have lost once for all our naïve acceptance of experience and its panoramic change as ultimately constituting the nature of things. Nevertheless, there would seem to be a need for something to take the place of what was lost. Purely formal principles like inference and the laws of thought may still appear too simple, remote, and abstract to account for the rich variety and complexity of our world. How, for

instance, shall we classify things like hunger and thirst, birth and death, equality and inequality? These taken *prima facie* are certainly not inferences; yet they bring us to a stand, while criticism dies in our throats before the impossibility of eluding recognition of their existence. Things like this, we say to ourselves, are what go to make up the world; after all is said and done, however it may be organized and logically hung together, what goes into it as its real constituents are facts and relations.

Yet facts and relations are ambiguous terms, and when accorded inspection seem to cover widely different meanings. To get at their essential significance, therefore, and to set them in harmony with other notions that we have accepted, may prove a task as difficult as it is important. What, in the broadest usage, can be said to be the characteristics of facts? An answer not infrequently met with is that they are those objects of experience marked preëminently by immediacy and independence. Thus, a fact is something so egregiously there that you cannot draw breath to question it; while, at the same time, it remains as brutally

unregarding of your wishes as an avalanche or a steam-roller that leaves everything crushed in its wake. A toothache, we say, is like that; absolutely immediate, absolutely beyond our power of direct control, hence a fact. And the same may perhaps be said of the state of affairs expressed in the propositions "New York is on the opposite coast from San Francisco" or "Heated metals expand."

But all such things as facts, it will be noted, are defined with reference to a subject. To be immediate and independent they require to be actually or possibly experienced by somebody. Yet this very correlativity to a subject, though essential to their nature as facts, militates against the fullness of their immediacy and independence. For facts, being apparently always relative to minds, are from the outset stamped with their organization; hence, it is impossible that they should enter experience as wholly underived and unconditioned, as so many irresistible shocks or concussions unenlightened by theory. From the moment of their first appearance, facts are categorized and mediated. A pure fact, if such a thing could be,

would hardly have its existence admitted; lacking all theoretical framework, with no stage set for its reception, it would be like Locke's substance: an "I know not what"—to all intents and purposes a mere nothing. Accordingly, we are forced to retrench on our earlier description of facts to an extent, and to describe them not as totally unmediated and independent data, but as whatever is, relatively speaking, accepted passively and uncritically in experience and as nearly as possible as given. The outstanding feature of facts, on this account, is at once the bludgeoning stroke dealt the mind and the mind's inability to resist it. In such experiences, we are struck down by main force both physically and mentally. Their towering presence robs us of the power of reflection; we have no breath left for question or criticism; we no longer act but are acted upon.

One great source of fallacy with regard to facts is the belief that they cannot deceive us, or that, if they do, the very deception constitutes an additional fact. The inconsistency here is patent: as if we said that a man cannot grow six heads, but that, if he does, they must all be

counted. The reason why facts are held to be veridical and impossible of deception is obvious. Since they dictate their own terms to us, while we in respect of them remain passive and helpless, we can hardly do otherwise than take them as whatever they declare themselves to be. If their face value changes, our concern (it is said) is not to question how these different appearances can be consistent with one another, but rather to acknowledge the new phenomena as so many additional and equally ultimate facts. Actually, however, it proves impossible to accept all states of affairs that claim to be facts as equally authentic. Too much confusion results; and the fact of error (one of the most overmastering of facts) would be thereby totally denied. Failure to allow for error is, indeed, the crucial error of panobjectivism. For where every appearance is accepted as undeniable and as equally real with every other, the laws of reason and the distinction between truth and falsity cease to operate. If such a world were, reason were not. But, since the human mind is committed on the contrary to rational assumptions, the existence of error may

be taken as pointing to the impossibility of a world of pure fact.

Not every appearance, we may conclude, is to be accepted at its face value; nor does the immediate irresistibility of an object prove the object genuine. Everywhere throughout concrete experience error is possible; and there is nowhere a fact so brutal that its reality is beyond risk of challenge.

To this statement, of course, the empiricist may be expected to take exception. "Nothing," he will urge, "is so perfectly incontestable as a fact. How can I possibly question, for instance, the pain of my toothache, or that cherries are red, or that bodies gravitate?" The answer which rationalism may make to him is more fully developed in another place.[1] But in essentials the reply to the empiricist takes the form of an analysis of the discrepancies between our *sensations* and our *interpretations* of them. Take the redness of cherries, for example. Ordinarily, of course, we uncritically suppose that when we think that we are perceiving red, we are beyond question perceiving just that

[1] Cf. ch. VIII, pp. 324-327.

color; in other words, that the *sensation* and the *interpretation which we give to the sensation* fully coincide. But, as a matter of fact, there is reason to doubt whether this is always so. If I am color-blind, I may believe that I perceive cherries as red when I am really sensing them as gray or something else. Again, if I utter the proposition about cherries at twilight, I may continue looking at them, continue to perceive them as red and to say that they are so, many minutes after they have ceased to yield me those sensations.

Yet, if the color that I experienced turns out to have been different from the color that I thought I experienced, what was the *fact* with which I came into indisputable contact? Once I admit, even in these borderline cases, that there may be unconscious variations between sense and meaning, how can I maintain that any given object is beyond the shadow of a doubt what it apparently declares itself to be? Moreover, when I turn to the record of history for a defence of facts, things are not much better. Even propositions about the most intransigent states of affairs seem to alter and conflict

from age to age. Not so long ago the geocentric view of the world was so much "hard fact"; while to the early Greeks it was apparently a fact that the rainbow had but three colors.[2]

Nevertheless, you may insist, there remains one absolutely unchallengeable fact at the heart of sense experience; for, although it is always possible that I may err as regards the "what," the "that" is indisputable. While I may be mistaken in my *interpretation* of any given sensum, at least the fact *that I experience what I experience* is beyond question. But even against the seemingly invulnerable certitude of the "that," rationalism is able to urge certain negative instances drawn from the borderlines of perception. Most of us can recall, for example, occasions when we have asked ourselves whether what appeared to be a barely discriminable pain was "real" or "imaginary"; in other words, whether we were really feeling anything or not. Again, we may have had the experience of listening to a train as it receded into the distance, and of noting that, in the diminishing series

[2] Müller, Max., *Lectures on the Origin and Growth of Religion*, p. 37.

of sounds, there occurred a point at which perceptibles merged into imperceptibles, certainty into uncertainty, until we could no longer tell whether we were listening to anything. Now, if we were asked at this point as to what constituted the core of "incontestable fact" in our experience, we should have to admit, ridiculously enough, that our sole certainty was our uncertainty that we were experiencing anything. For that matter, even if it should be granted that we experience whatever we experience, assurance on this score amounts to little more than a tautology. Such certitude regarding facts is formal only, showing perhaps that *a* is *a,* but shedding no light whatever upon their content.

Yet what if this formal identity were the core and solution of the whole matter? Perhaps we were wrong from the beginning in looking for certitude in the givenness and intransigency of facts. Whereas to the uncritical mind irresistible convincingness seems to attach to the particular content, perhaps it really belongs to the logical form as such. Certainly, when we try to conceive facts as pure realities *per se,* they lose all definite character. Somehow their alien

and inarticulate character prevents them from being assimilated to the rest of the universe. Moreover, complete insulation in their qualitative immediacy puts them in the condition of miracles, pure "gifts from above," lacking all possibility of being explained either through themselves or something else.

The untenability of this conception of facts is plainly shown when we seek to apply it to specific instances. To hold, for example, that the essence of water is something unique and irreducible is open to criticism. Chemical analysis has too well established it as a derivative of hydrogen and oxygen taken in certain amounts. But that such chemical deduction overrides its identity as water or explains it away, nobody seriously believes. Though its content is resolvable into simpler elements, its primary nature remains untouched. Thus, although no particular content can be assumed to be qualitatively ultimate and irreducible, the formal integrity of facts is preserved by showing what their present state is the result of, and, by exhibiting how they follow from a given combination of units. Reflection, in short, really maintains the iden-

tity of facts by revealing them as deductions. And, although this goal is far from being achieved in most branches of science, the general character of their progress seems toward the discovery of broad conceptual schema capable not only of including larger and larger tracts of phenomena, but from which further combinations of phenomena can be predicted and derived. The ideal of science seems to be the conversion of facts from the status of brute data to that of established instances of general laws. Even in those complex regions of experience in which laws are still largely unknown, facts are taken as "givens" only in the sense of being assumed as *points of departure* in the investigation, and not as in their essence opaque and uncognizable. For, since the human mind can work in only one direction at once, and since it must start somewhere, the terms with which it begins have to be adopted for the time being without explanation but on the supposition, however, that at another stage of the process they can be explained theoretically just as the others. Perhaps, indeed, nature will always retain an unexplored residuum

of phenomena; but at least we may assume that it harbors no problems in essence insoluble.

Thus, although truth is in facts, facts by themselves are not true. It is only, seemingly, when facts are comprehended as terms in a relation, as specimens of a class, or as members of a larger system, that their real function is revealed.

But already our story has outrun itself; and relations have been drawn in to supplement facts. Indeed, every account of facts has recourse in the end to principles of connection; for facts themselves are data to be used rather than agencies of organization. Accordingly, relations must be appealed to if facts are to be united into a world.

II: What are Relations?

A large measure of the analysis expended upon relations in historical philosophy has apparently failed to reveal their essential character. The fault seems to have arisen through the uncritical extension of the name "relation" to any term that refers in its primary signification to other terms, or that expresses any sort

of mediative activity or functioning. Thus, the name is applied loosely to any way of taking things together: a crowd, a yoke, a halter, swimming, rowing, fighting; all these are called "relations" in an unreflective sense. Such blanket usage, however, robs relations, as it seems, of all distinctive significance and results in the loss of their discriminating marks. In default of any clear indication of their fundamental dissimilarities, they have come to be dealt with after the same fashion as facts or empirical phenomena. That is, they are taken as part of *de facto* reality: both by the realists who conceive them as so many atomic givens or subsistents, and by the idealists who interpret relations as the feelings and impressions of activity experienced by a mind. But neither the idealistic account of relations as synthetic mental activities nor the realistic description of them as data independent of consciousness really succeeds, as it appears to us, in distinguishing relations from facts.

In themselves, the realists urge, relations are indefinable; and here, by waiving direct discussion of their nature, they partly evade their

critics. At the same time, according to their account, relations are characterized by a distinct mode of being known as "subsistence," which sets them apart from the "existence" of physical objects and sense-data. Besides the realm of existence or the space-time order in which objects change and interact, there is said to be a world of eternal, changeless essences, in which are housed all possible objects of discourse, including relations. But here a difficulty seems to arise. Although relations are held to "subsist," the realists emphatically deny that they possess a "substantive" character. In opposition to the old substantialistic treatment of relations based upon the common-sense view of the world as composed of "things" and their attributes, these modern thinkers are concerned to emphasize instead their *functional* character, and to point to the large number of transitive, asymmetrical relations so important in science, which seemingly resist all attempts to reduce them to the status of adjectives or qualities of a subject. Precisely here, however, the critic of realism might well call their attention to the fact that, while disclaiming the substance-at-

tribute view of relations, they are themselves utilizing the same conception in taking relations as "subsistents"; since subsistence, upon examination, proves to mean nothing but substantial existence in attenuated form—that is, in abstraction from its spatio-temporal aspects. The contrast between the ideal and the real is something far more radical than any mere difference between modes of being of divergent densities, or the presence or absence of space and time. A distinction between the self-dependent and the externally dependent would perhaps be more to the point in this connection. But, without pursuing this farther, one may still assert that, in so far as the realist regards relations as indefinable givens or subsistents, he continues to envisage them as substantial and existential rather than ideal.

Similarly mistaken notions of relations are to be found among idealists, who have, if anything, an even harder task in differentiating them from sensible facts. For, since idealists hold both relations and sense-data to be dependent upon their presence to a mind, they are obliged to find some criterion within mental ex-

perience itself by which to distinguish the one from the other. Yet all too frequently in experience they appear virtually indistinguishable. Some idea of the difficulty of finding such a characteristic mark for relations is obtainable by anyone who undertakes to analyze, for instance, his awareness of depth or of motion. Even in the most rudimentary perception of this sort, there is present, in addition to the sense-qualities, some discrimination of "this" and "that," some contrast of "core" and "fringe." To have a sensation at all involves experiencing the sensa in some order or arrangement, however indefinite—an arrangement which is so inextricably interwoven with, and impressed upon, the data that it is often scarcely separable or distinguishable from the sensuous manifold, being itself also apparently presented in organic terms. But since sense-data are accepted on all hands as primarily "given" in experience, and since, in a wide range of spatio-temporal situations, it appears impossible to analyze out or satisfactorily to discriminate any factors over and above the sensuous, there seems no recourse save to accept

the total situation as broadly given and presentational.

Nor is a happier solution to be found in the idealist's attempt to discover the differentia of relations in the active, synthetic participation of mind in certain phases of experience, as contrasted with the passivity that characterizes sense-impressions. Although the distinction is no doubt of relative importance, it is, after all, merely psychological and not of a character to relieve relations of the charge of existentiality. Plainly, the mere removal of relations from the passive to the active side of mind is not in itself sufficient to free them from a conception in principle *de facto*. Moreover, modern analysis has come to recognize that the mind and its objects cannot be strictly divided into separate zones of activity and passivity; but, on the contrary, every act and content seems to include both. No sensation, for instance, is a wholly passive experience, but involves active adjustment and adaptation to the given in the same sense as (though to a lesser degree than) conceptual activity.

The notable similarity between sense-data

and relations, admitted by realists as well as by idealists, is that both arise on the occasion of a stimulus *ab extra:* the one in the world of sense, the other in the so-called conceptual world. Both are provoked or touched off, as it were, by some excitation outside our direct conscious control or powers. When I perceive two things as equal, for example, this equality seems to force itself upon me independently of my desire or will, just as certainly as a shrill sound or a bright color intrudes itself upon attention irrespective of my wishes or direct control. Moreover, since the totality of experience is of a piece, and since the sensations or foundation-stones of it are "given" in such a manner as to be hardly distinguishable from the relations that serve as the mortar, the whole would seem throughout to have the same substantive character. But, if this be so, neither the idealistic nor the realistic account really succeeds in freeing relations from the charge of existentiality.

III: Relations Are not Facts, Presuppositions of Experience, or Class Concepts

Whatever else they may be, it is clear that relations cannot be satisfactorily described as mere facts. For facts, as we have seen, are relatively "given" entities lacking within them the principle of their being, and in their nature *data to be used* rather than organizing operations. Hence, facts cannot really unite facts; and the attempt to make them do so only gives rise to an indefinite process. Thus, if, between any two facts to be related, a third fact is posited *as their relation,* it will be found that nothing is accomplished thereby save the addition of a new fact to the original terms. So we are obliged to hunt anew for a relation, this time to connect the so-called relational fact to the original ones; yet, since this relation must in turn prove factual, we face the prospect of repeating the process *ad indefinitum.* And, if it be urged in objection that the mere presence of the infinite process in a relation need not destroy its efficacy, we may reply that at least we do not find in our thinking any tendency to interpolate an

endless series of intermediaries between terms in order to get them together. Such a procedure would seem to be animated by the contradictory notion that the way to bring things together is to separate them as much as possible.

Nor can relations be construed any more satisfactorily simply as presuppositions of experience. Experience is only possible, in the opinion of certain philosophers, because it rests upon synthetic forms or relational frameworks which introduce order into its structure. The sole proof of the reality of these relations, however, comes from experience itself. Without postulating their existence, it is apparently impossible to see how our organized world and the systematic body of physical science could have arisen. Such is the point of view of Kantianism, a position not devoid of difficulties. In the first place, relations are adduced as the unifying elements of experience solely on the testimony of experience, and in so far by a circular process. Moreover, the mere prevalence of such principles in experience is scarcely warrant for believing that they are peculiarly constitutive or fundamental in our world. Limited to the

circle of experience in our knowledge, as this philosophy supposes us to be, we have no way of distinguishing between principles that are essential and those that are merely accidental. Such a view inevitably makes relations mere adjectives or appendages of the empirical world, and, while claiming to define them as "presuppositions" and hence as independent of phenomena, is nevertheless obliged to admit that they hold only between phenomena. Thus, relations are left in the equivocal position of being recognized as inseparable from experience yet such that experience can never theoretically justify their use.

The treatment of relations as class-concepts is open to criticism no less serious. Unlike true universals, these generalizations appear to be sustained by no inner necessity but to subsist in accordance with a purely factual mode of being. Like sense-data, they simply "occur" without shedding any explanatory light upon themselves and without revealing to the mind any standard of comparison by which experience may be evaluated. Inquiry into the nature of these concepts shows them to be derived from

given, concrete reality. They are constructed by a process of separating out some common, recurrent feature from a number of experiences; so that eventually, through repetition of the procedure, there arise conceptual divisions corresponding to the factual similarities running through particular things. Just as we come to know redness by abstracting out a specific quality from many similar sense-experiences and considering it apart from its original setting, so likewise, it may be said, we reach a generalized notion of goodness by isolating a common quality found shared in various states of affairs that have power to arouse in us what may be called agreeable feeling in a wide sense. This process of abstraction is partially traceable in the history of language; and instances where it is still plainly going on may be found among primitive peoples: for example, in the dialects of certain African tribes where the word used to describe the position "in" is the same as that for "stomach," and where that for "high" is the same as "neck," and so forth, thus showing the difficulty experienced by the primitive mind in trying to conceive these "re-

lations" in detachment from particular things. Gradually, through long practice, however, the significant common element in a multitude of cases is clarified and abstracted, the irrelevant details fade out, and the arrangement or position of entities with respect to each other stands forth as the essence of a concept.

Class concepts, then, no less than sensations and feelings, take their rise from that indefinite manifold that we posit as the source of immediate experience. Yet, since mediation everywhere accompanies immediacy, general notions and sensations are not taken as wholly concrete and underived, but are rather understood as more or less abstract signs or representations that point back to the manifold. Because both involve abstraction, both inevitably refer beyond themselves to the source from which they emerged, and in so far involve external reference.[3] The best illustration perhaps of such ref-

[3] Instead of referring to the *terminus a quo,* genuine universals, it should be remembered, indicate a *terminus ad quem;* that is, in place of representing by a kind of composite picture the manifold whence they emerged, universals offer the mind a standard or determinant by which to evaluate sensuous particulars, but which claims itself to be separate from, and fundamental to, the particulars.

erence is to be had from the simple analysis of experience itself—in the fact that we never, in ordinary life, sense our sensations or conceive our concepts; but, on the contrary, sense or conceive some object which we assume to be distinct from, and the occasion of, our mental state.[4] The sensation or concept serves instead only as an interpreter that stands between the subject and the state of affairs to which reference is made, a transparent medium unnoted unless our attention is especially called to it, like glasses that correct an astigmatism but of whose reorganizing work we are usually unaware. Thus, we never experience a sensation that is brown, resinous, and sticky, but have a sensation *of* a brown, resinous, sticky object; nor do we have a red, equal, or just concept, but rather notions *of* redness, equality, or justice; similarly, we do not apprehend an "and" or an "of" concept, but concepts of "and-ness" or "of-ness."

But, although class concepts no less than sen-

[4] In cases where a concept is itself made an object of conception (as where I think, not of equality, but of my idea of equality), it is plain that the concept has ceased to serve its function as medium or intermediary and is being taken instead simply as a psychically existent fact.

sations retain an external reference, mistaken notions in regard to them are rather prevalent. Especially is this the case with respect to ideas like "and," "of," "before." There is, for instance, a wide-spread belief that such concepts themselves couple together objects in the objective manifold. Their status as abstract signs or representations being forgotten, it is imagined that in some mysterious manner they can enter into and affect concrete states of affairs. Yet to imagine that they do so, is as absurd as to believe the concept "sticky" capable of gluing together physical objects, or that the notion of "harness" is enough to hold a horse between the shafts. Since we do not expect our sensations to sense or to be sensible, surely we should not expect mental representations of "relations" to relate.

Even where this mistake is avoided, another is likely to appear. Although forced to admit that concepts themselves do not relate, some will probably maintain that there are objective states of affairs, to which the concepts refer, which function as binding links in the external order. Although granting that we cannot im-

agine how the concepts "of" and "and" should themselves bridge chasms in the concrete world, they will insist that the "of-ness" and "and-ness" which they indicate, are real subsistent connectives in the objective order. Yet, here again, there is a subtle illusion. What is overlooked is the fact that "of-ness" and "and-ness" are to a seemingly limitless extent human constructions, referring to the ordering and grouping of data as they appear to our accidental, particular points of view, and that they cannot therefore claim to subsist as connectives in a distinct and separate world. How completely such "relations" are the product of man-made abstraction may be shown by asking oneself what difference it would make to our world if certain features of experience were altered. Suppose, for instance, we should suddenly find ourselves to be dwellers in flatland, beings sensitive only to two-dimensional space: in that case, there seems reason to believe all the "ups," "downs," "aboves," "belows," and so forth, would, on the instant, be annihilated. Or again, if we should lose the use of our senses all at once, presumably such concepts as "brighter-than," "sweeter-

than," "one-inch-taller," etc., would vanish from the world. But once it is apprehended that entities like "on-ness" and "of-ness" are only composite photographs, as it were, of the grouping and positions of particular objects with respect to one another relative to us, it becomes clear that such entities can exert no force to bind things together, but, on the contrary, far from being true connectives, are merely "relatives" or signs depicting the location in space, time, or some other medium of certain concrete states of affairs, which by reason of their generality hold of a wide range of cases with equal success.

IV: Relations Involve Terms and a Necessary Connection between Them

But, having avoided some possible misconceptions, what in a more positive, discriminating sense shall we say is the nature of relations? The first point to be noted when we sift our knowledge of them is, of course, that they involve terms. Relations are present only when two or more terms are taken together. They must, in

short, relate something. The nature of the terms need not be dwelt on here; presumably data of any kind will serve, though the manner of their incorporation is not yet clear. Secondly, it is generally agreed that a relation involves some sort of connection, link, or bridge between its terms. Here we reach the heart of the problem. Whenever relations are considered in abstraction, this characteristic of being an intervening tie is taken to be what especially distinguishes them. The fact here indicated, that a relation always stands *between* the terms related, is important, since it seems to suggest that the relation itself (no less than its terms) is a discrete, particular entity. But if a relation is really such a distinct intermediary link between things, questions properly arise as to what this link consists of, and as to how and by what means this intermediary can itself be connected with each of its terms.

Yet these questions, when clearly put, give rise to problems of so grave a nature that the answer to them has led historical philosophy more than once to a denial of relations altogether. Since the time of Locke and Hume,

in fact, many persons have come to the conclusion that we have no actual impression or immediate representation of relations in experience whatever. On the contrary, it is said, all we detect, properly speaking, are so many loose and separate qualities, events, or terms, which, though they occur in certain peculiar aggregates, seem never to register any concretely observable tie or connection as holding among them. Thus, although the sense-qualities hot, round, bright, etc., appear associated together in a certain way in experience, so that we customarily refer to this aggregate by a name as "the sun," we can never, as a matter of fact, picture to ourselves what it is that unites these qualities into one object. Presumably, all we can say is that, owing to their frequent conjunction in experience, we have come to have a certain "feeling of relation" or expectation of their being associated in the future, without, however, being able to perceive any sort of actual adhesive force or sticking-plaster joining them together. The climax of this view is to be found in the writings of Hume who, reasoning from the absence of any distinct "impression"

of relations to their unreality, declared that "we have no idea of connection at all and that these words are absolutely without meaning."

Having arrived at the apparent downfall of relations, shall we finally abandon them as the empiricist urges and as the logic of the infinite regress strongly invites? Certainly the arguments in favor of doing so are strong. It is just possible, however, that, while renouncing relations in the sense of distinct mediatory entities, we may yet discover a more defensible meaning of the term. Surely to relations under some signification or other must be imputed the pattern and organization of our world. Even the most convinced skeptic has to admit that he has "feelings of relation" and that, despite his best agnostical efforts, he cannot wholly rescind belief in them. Ultimately, indeed, it seems impossible to doubt that there are relations in the universe; for, in the very effort to deny them, recourse must be had to grounds for their denial; whereas the acceptance of grounds as valid implies the reality of necessary connections, a nexus of relations. These relations, however operative in abstract reason-

ing, appear upon reflection very different from the factual intermediary entities which we have hitherto conceived as synonymous with relations. Whereas rational connections seem to issue as inevitable logical consequents from the combination of certain terms with which they enjoy an integral validity, the former interpolated links were taken as *standing between* their terms and as adjusted to them in some spatial, extraneous way in accordance with a mode of being or subsistence.

To Hume, indeed, belongs the credit of first clearly perceiving that if relations are really to join terms together they must be of the nature of necessary connections; although he was mistaken in his further assumption that such necessary connections must infallibly register some image or "impression" of themselves upon the mind. Failing to find such an image recorded, he drew the unwarranted conclusion that relations are illusory and unreal, overlooking the fact that reasoning (including his own reasoning in reaching his conclusions) proceeds by the use of just such connections which, though not pictorially representable—because of their

strictly formal character—are nevertheless assumed to express an indefeasible synthetic compulsion. To us, indeed, it would seem that a bond of this sort is ultimately the only genuine type of relation possible. For in order really to relate, relations must possess a coerciveness and cohesiveness that is beyond question. And this determination cannot reside in some blunt natural force, since facts, as we have seen, are impotent as connectives. Nor can a relation be a mere conceptual hooking-device that is presented to the mind for acceptance or rejection at a look; on the contrary, relations to be comprehended require to be thought through, and thus involve an intellectual operation. That is, true relations, upon which we rely in thought, persuade our minds but do not impress them *ad baculum*. In this, as we have said before, they exhibit their genuine independence; since, instead of imposing themselves bluntly upon the mind like brute facts (irrespective of our understanding of them), they lead the mind to accept them on its own conditions. In brief, true relations, as it seems to us, must be necessary connections, possessed of some inherent rational

determination. For so far as we can see, the only bonds of linkage admitted on all hands to be reliable are those underlying reflection which connect the sequence of criticisms even in the critical process itself.

V: What Constitutes Necessary Connection

Having arrived at the conclusion that relations are logically necessary connections, our task is to elucidate the nature of these if possible. At the present time, it is customary in many quarters to explain necessary connections almost entirely in terms of implication. Logic and mathematics are conceived as the sciences of implication; throughout their operations, conclusions are said to be drawn from assumed premises by means of implication acting as the fundamental type of formal necessity. Yet there is, as we soon discover, no single sacrosanct meaning of the term "implies" agreed upon. Instead, alternative meanings are apparently adopted as the bases for different systems of logic.[5] Thus, the system of Russell and White-

[5] For a discussion of different possible systems of logic and

head rests upon what is called "material implication"; here "*p* materially implies *q*" means "It is false that *p* is true and *q* is false."[6] The calculus developed on this basis has to do with the truth-values of propositions, taking the latter fundamentally in extension and envisaging existentially the inclusion and exclusion of classes. On the other hand, systems like that of Lewis make use of what is called "strict implication." The strict implication "*p* implies *q*" means " 'It is impossible that *p* be true and *q* false,' or '*p* is inconsistent with the denial of *q*.' "[7] Utilizing this conception, a logical system is generated covering, it is claimed, both extension and intension; and thus including (in addition to the realm of existence) the regions of the possible, the impossible, and the necessary. It would seem, then, judging by such systems, that the definition of the term "implies" is more or less of an arbitrary matter;

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meanings of implication, cf., Weiss, P., "Relativity in Logic," *The Monist,* 1928, XXXVIII, p. 536 ff.

[6] Lewis, C. I., *A Survey of Symbolic Logic,* p. 326. Whitehead, A., and Russell, B., *Principia Mathematica,* 2nd ed., vol. I, p. 94.

[7] Lewis, C. I., *op. cit.,* p. 332.

and we are left with the question on our hands whether all necessary connections of a logical sort depend upon acts of free postulation and whether, if they turn on little changes in the meaning of "implies," we may not expect as many revolutionary and conflicting logics to emerge as did geometries following alterations in the postulate of parallels.

But here, even at the risk of error, it becomes necessary for us to state our opinion that the validity of relations does not in fact depend upon the particular meaning that the reasoner happens to give the term "implies." True enough, thought imputes connection to its terms and apparently ascribes this concatenation to logical necessity. But, when one asks what constitutes the necessity of a system, it becomes obvious, I think, that this is not anything expressly contained in the definitions and postulates of the system itself. Nevertheless, all thinking rests upon certain assumptions. Speaking pragmatically, the mind, in committing itself to the struggle with its surroundings, does so on certain terms. Or, as we might say, reason, in laying hold of its world, is governed

by the laws of its own self-determination. In the first place, it recognizes its freedom to pick and choose any definitions of "implication," "consistency," and so on, or any postulates whatsoever for incorporation into a given system. In other words, the mind posits its right to take anything that it pleases in the way of marks or counters, and to attach to them any meanings or symbolic significances that it likes. Thus, thinking in its construction is very much like a game of hop-scotch. You can have, in your checker-board, any number of squares you wish, and you can define as you please the rules of the game: that is whether the players shall hobble on two legs or hop on one, whether there shall be free bases or "King's X," etc. But, when you have allowed arbitrary convention free play up to a certain point, an element of rigidity enters. Since the reasoner in constructing the operations of the system, agrees to them himself, it becomes incumbent upon him to conform to the rules he has laid down. In other words, having chosen the rules of his game, the player is obliged, if he would play the game at all, to abide by them. Did we say

"obliged"? Immediately, there are those who will wish to know what can be meant here by the term "obligation." And the answer is, as it seems to me, that an obligation arises from the fact that otherwise the system cannot be tested for truth or falsity on its own terms. All thinking, as was said, aims at some sort of truth, and admits therefore of a criterion which is, in a fashion, laid down by itself. On its own assumptions, the system is not valid unless it possesses a kind of operational integrity or self-consistency. In other words, there must be congruence between its promises and its performance, between what it says it will do in its postulates and what it actually does in its manipulations.

Yet, in the very recognition of the necessity of "playing the game" and "sticking to your postulates," if there is to be any articulate system or discourse whatever, is implied, as it appears to me, the outline of a logic that is fundamental and common to all special alternative logics; for whatever your logical system is, if it is to have operational integrity, it must hold strictly to the rules it has laid down for itself.

Otherwise, in what could its validity consist, if not in the fulfilment of its own conditions, since it recognizes no external or superior authority? But assuredly, if "sticking to the rules of your game" means anything, it means that terms must be used, throughout the operations of the system, strictly in the same senses agreed upon in the premises. And this is obviously equivalent, first of all, to the law of identity. *X*, if defined as *X*, must not be used for *Y*, *Z*, or *M*, unless special provision for such substitution is included in the postulates and definitions. Again, specificity involves exclusion: definition of our chosen rule (*X* is *X*) denies and excludes all other rules (such as *X* is *non-X*). Furthermore, insight into the identity of any given rule (*X* is *X*) tells us that what-is-excluded-from-its-identity is *all there is;* in other words, it conveys the mutual exhaustiveness of *X* and *non-X*. Again, any application of these rules must involve such operations as the assigning of values to variables and the subsumption of cases under general laws.

Yet, suppose that you decide to make the rule of consistency in such a system that of

inconsistency; in that case, at least, you probably imagine, you will be free from the criteria of the laws of thought and the so-called general logic. Yet, even here, if your system is to measure up to its own terms, you must play the game, meticulously avoiding all concepts that do not conflict with one another and striving to be consistent in your inconsistency. Thus, even where these rules of consistency are denied in the premises from which we reason, they none the less operate as the principles in accordance with which we reason.

Any system with operative self-consistency, in which conclusions are drawn from assumed premises in accordance with preëstablished rules, may be called, therefore, in our view, a valid logical system. Even the bizarre reasonings of the insane, who attempt to justify their obsessions by elaborate trains of argument, may conceivably be admitted to this class. Thus, ample room is accorded the claims of alternative logics even, as we have seen, to the extent that what is termed consistency by definition within one system may be defined as inconsistency within another. In other words, although

logical systems, just as geometries or algebras, may spring from contradictory postulates, this does not destroy the inner coherence of their respective worlds or prevent both opposites from being valid, provided that, on their suppositions, they develop no internal inconsistencies. And the fact that this is so is due, in our opinion, to the operation of certain universal principles as a ground-work of all particular logics, whose principles (like those of identity, noncontradiction, and subsumption) are involved in the very nature of thought and action whereever man chooses to lay down a set of rules for himself (no matter what they are), and, having done so, to "stick by" them and "play the game."

Moreover, wherever a set of assumptions is manipulated so that they hang together in this way, certain products are to be found in the resultant that were by no means obviously inserted in the premises. Ultimately, the inferential process underlying these systems is something more than a mere play of classifications, substitutions, exclusions, etc., all of which might be better done by machinery; for it essentially in-

volves "going from" something to something else, and the claim that this process establishes or "proves" the validity of some new combination. The necessity of a given system comes, then, apparently not from any special meaning that we attach to the term "implies" or from the particular postulates chosen for the system, but emerges in the course of the operations as something distinct from these assumptions, although generated through reasoning in accordance with them. And, since these postulates may differ widely in different systems, while only the requirements of "playing the game" remain common to all, it must be concluded that the element of necessary connection is to be looked for wholly in the latter.

Furthermore, wherever there is necessary connection, there is inference; for a relation is always the expression of comparison, the notation of identities and differences among members of a group. One star cannot be brighter than another, one event later than another, unless there is some basis of similarity between the two, making possible their reference to a common standard by means of which their positions

may be defined with respect to one another.

Exception, however, is sure to be taken to this account. If relations are ultimately reducible to inferences, it will be said, all distinctions and differentiæ of their specific types will be lost; or, at any rate, their peculiarities of "in-ness," "by-ness," "before-ness" will be held to have no connection with their function as relations. To this criticism, in our opinion, at least two answers may be made. First, the history of the subject has shown more or less conclusively that relations, when taken apart from inference and treated as forces, substances, or qualities, fail to relate. Second, there seems no good reason why admission of the inferential nature of relations should be irreconcilable with differences in their content; or why the necessary connections embodied in them need be affected by their expression under the different categories of space, time, substance, or causality.

The first of these points has, I take it, already been made. Expressions like "of," "by," "before," "ancestor of," when taken apart from inference are nothing more than *relatives,* terms whose primary significance is derived from their

position with reference to other entities external to themselves. Far from being genuine connectives, they are mere epithets descriptive of the spatial, temporal, and factual collocation of things as we happen to apprehend them. At this level, the world appears as a kaleidoscopic jumble of existential bits; its so-called relations being no more than syncategorematic concepts which register the bearing of these bits upon one another. Thus, whenever a class-concept is taken as a mere common quality abstracted from a group of particulars, it remains itself a vague, generalized particular on a level with other particulars. At most, it functions only as a sign or indicator pointing to the concrete individuals, yet without connecting anything, since there is obviously no synthetic force in mere reference. So long as the would-be connective is conceived as nothing but a detached part or selected feature of a group, it remains a mere external supernumerary, wholly failing in its appointed task. In the actual process of reasoning, however, we come to observe that the concept often functions as a law or explanatory ground of the particulars in a cer-

tain respect; that is to say, in concrete reflection the specific terms are not usually taken as so many flatly given, uninterpreted data, but as exemplifications of universals which underlie the facts and account for their being as they are. But, once the idea of the concept as a detached feature of a group of objects is discarded, it is replaced by the notion of some common ground of unity or rule that runs through the members uniting them, a notion essentially indistinguishable from that of inference. Wherever any items can be taken as a collection, there, we may say, a relation is to be found; for there the members of a plurality are assimilated to some common basis of reference and thence related to one another.

The second point to be maintained—namely, that the inferential view of relations need not fail to take account of distinctions of type, but may, on the contrary, be capable of differentiating exactly in comparisons of quantity and degree—is a matter to be dealt with in more detail in the following section. For the present, let us merely call attention to one fact: that relations, like everything else, have two aspects—

the quantitative and the qualitative. Now, in so far as sole emphasis is placed on this qualitative and exclusive phase of relations, no articulate or exact account can be given of them. Whether the terms dealt with express degrees of a single sense-quality (such as "sweeter," "brighter," "louder") or spatio-temporal qualities attributable to several senses like sight and touch (e.g., "in," "on," "before," "the right of"), in either case they indicate, in so far as their meaning is purely qualitative, something private, particular, non-relational, and incommunicable in the content. Though the immediate character of an experience, as experienced, may perhaps be pointed to by a name, it can never be expressed in terms of a comparison or scale of computable likenesses and differences. This restriction, however, is in no way peculiar to relations, but springs rather from the unique, untranslatable nature of all qualities. But, if this be so, differentiation among relations has ultimately to be expressed in terms of quantity and form, since it is only through this aspect, if we are not mistaken, that relations are related and in turn relate.

VI: Answers to Possible Objections

The view of the preceding sections may perhaps best be developed further by trying to meet some objections. Relations have been represented in the foregoing paragraphs as formal, quantitative, and inferential in structure. Now, exception may be taken to our description by certain logicians, not only on the ground that it ignores the atomic character of relations, but also that it fails to do justice to non-syllogistic types of relation. The main charge brought against traditional formal logic in recent years has been that, since it reduces all propositions to the subject-predicate type, it cannot possibly take account of other kinds of relations. Aristotelian logic, as is known, arose as an attempt to describe the world of things and their properties; and, as was quite natural in such a case, it came to interpret all propositions on the analogy of substances and their attributes, although relations on this view had to be depressed to the status of mere adjectives or qualities of the terms which they claimed to relate. Now, quite apart from the question whether an infinite process is generated where a connective is treated

as a mere property of its terms, a distinct problem here confronts the logician in that there appear to be certain relations that cannot be reduced to the status of mere attributes of a subject.

Among the most interesting of these so-called non-syllogistic relations is the group which may be known as asymmetricals, comprising such concepts as "ancestor of," "before," "after," "greater than," "one inch taller," and so forth. The peculiarity of this class is that if the relationship holds between x and y, it can, under no circumstances, hold between y and x. There are, moreover, many indisputably valid chains of reasoning of the sort: "x is less than y, and y is less than z; therefore x is less than z." Practically, wherever notions of serial order, space, time, number, or differences of degree or magnitude are used, these asymmetricals appear to furnish the basis for trains of reflection of a kind that cannot be reduced to a group of simple propositions in which the predicates are attached to the subjects merely by the copula "is." Accordingly, there are many who would almost go the length of denying the name "rela-

tion" to the subject-predicate construction altogether, maintaining, on the contrary, that the attribution of qualities to substances spans no chasms in thought or reality, but rather widens the gulf by opening up an endless regress.[8] Equally ridiculous to them appears the tendency, in syllogistic thought, to translate all predications into adjectives of their subject-terms: as if "to the right of" were a property of this apple tree, or "is less than" were an attribute of the planet Mercury. Instead, the term "relation," in their opinion, should be used primarily to denote asymmetrical constructions such as are found at the basis of the physical sciences in spatio-temporal and mathematical measurements marking differences in quantity and degree, all of which seem, at first sight at any rate, irreducible to the subject-predicate form.

Before assenting unqualifiedly to current

[8] This line of criticism is already too familiar to need amplification. If predicates are related to their subjects, how are they attached? Certainly not by pure identity; for if this were the case "sugar" and "sweetness" would be absolutely indistinguishable. But if not related by pure identity, some supernumerary would have to be inserted between them, which in turn would demand another, and so on *ad infinitum*.

opinion in the matter, however, it appears desirable to examine some examples of this type of asymmetrical reasoning in detail to see whether they really differ so sharply and irrevocably from the traditional syllogistic type. To our way of thinking, it is hard to believe that deduction can be bifurcated sharply into syllogistic and non-syllogistic relations. Reasoning would seem to be too much of a piece. Offhand, we should be inclined to lay the difficulty in the doctrine of relations to the fact that they are atomistically treated (as parts of propositions instead of as necessary connections between propositions) rather than to any inherent dualism in their nature.

To begin with, take the so-called *a fortiori* argument: "*A* is less than *B*, and *B* is less than *C*; therefore *A* is less than *C*." This can be formally expressed as follows:

All *A* is *B* (and some *B* is not *A*)
All *B* is *C* (and some *C* is not *B*)
Hence all *A* is *C* (and some *C* is not *A*).[9]

Next, let us compare this reasoning with a

[9] De Morgan, A., *Formal Logic or the Calculus of Inference*, p. 22.

typical syllogism, such as 'All *A* is *B*, and all *B* is *C*; therefore all *A* is *C*.' Ordinarily, of course, the fact is disregarded that each judgment in the syllogism, if its meaning were more fully expressed, would also take the form of a complex proposition. Because common syllogistic reasoning emphasizes the identity of the middle terms rather than the quantification of the predicates, and because its purpose is primarily to arrive at a product qualitatively distinct from the elements that enter into it rather than quantitatively, we usually do not take the trouble to state the meaning of its propositions with complete explicitness. More fully expressed, a syllogism of this familiar *AAA* type would read:

All *A* is *B* (and some *B* is not *A*)
All *B* is *C* (and some *C* is not *B*)
Hence all *A* is *C* (and some *C* is not *A*).[10]

[10] Of course, definition and other rare instances where the subject and predicate terms exactly coincide have to be taken account of as a limiting case. In any event, all predicates are assumed to possess definite quantity, even though this may be left unexpressed in ordinary speech. In order to cover all possibilities, the syllogistic argument may be stated, if desired, as follows:

But once the meaning of the ordinary syllogism is stated precisely and explicitly as above, it is seen to correspond closely to the structure of the so-called asyllogistic argument. As will be observed, the chief difference between the examples of syllogistic and non-syllogistic inference, is that whereas the one ordinarily neglects a part of the meaning of each proposition and includes it only, as it were, as a suppressed immediate inference, the other through quantification of the predicate (as well as of the subject) expresses the full bearing of each upon the other in clear and concise form. But aside from this, their structure is markedly similar. The *nervus probandi* of both, for instance, is a three-term argument like the classical first figure syllogism in appearance, in which the advance is made by means of a middle term. Again, both comparisons use marks of quantity and quality; while each proposition of the fundamental syllogism into which both may be analyzed is accompanied by an immediate in-

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All *A* is *B* (and either all *B* is *A* or some *B* is not *A*)
All *B* is *C* (and either all *C* is *B* or some *C* is not *B*)
Hence all *A* is *C* (and either all *C* is *A* or some *C* is not *A*).

ference as an aid in expressing the distribution (or lack of distribution) of the predicate. In any case, the predicate is assumed to possess a definite quantity, even though it may be left explicitly undetermined. Considering all these similarities, there seems little chance of maintaining that a fundamental bifurcation subsists between deduction of the so-called asyllogistic and syllogistic types to the extent that it is impossible to reduce the one to the other.[11] At the same time, it must be admitted that, wherever

[11] It is true enough, of course, that the example of asymmetrical relations cited above cannot be directly reduced in its original wording to a simple syllogism. For when this is attempted (Cf. J. N. Keynes' *Formal Logic,* p. 385) we find that we are left with a simple syllogism plus an immediate inference on our hands, an inference which seems still to be of a character that cannot be admitted as valid so long as we recognize only the copula "is." If we try to reduce the argument to the form "Whatever is less than *B* is less than *C, A* is less than *B;* therefore *A* is less than *C,*" a *petitio* is plainly involved; since we have added to the original statement "*B* is less than *C*" the assumption of the principle of quantity that "Whatever is less than a given thing is also less than anything which is greater than this given thing." And this immediate inference, tacitly introduced, is really equivalent to the assumption of the very relationship it is desired to prove. On the other hand, when taken exactly in its original wording, the inference "*A* is less than *B; B* is less than *C;* hence *A* is less than *C*" plainly involves four (instead of three) terms and lacks a middle term.

our aim is to emphasize differences in quantity or measurement, non-syllogistic modes of expression are best, since these latter, by carrying the quantification of both the subject and predicate to various lengths of mathematical exactness, enable thought to state the precise degree of difference between terms and to achieve a conciseness and economy of expression far in excess of the traditional qualitative syllogism. By trading primarily upon the quantitative aspect of reflection, the argument, though sacrificing something on the qualitative side, gains immensely in the stride of its advance. At the same time, throughout the operations of geometry, algebra, and physics, no less than in the sorites of biological classifications, one still recognizes the use of that indispensable tool of the intellect to which even William James paid homage, admitting that "once we know that whatever is of a kind is also of that kind's kind, we can travel through the universe as if with seven-league boots." [12]

But, if it be granted that asymmetrical quantitative arguments are rooted in the same gener-

[12] *Pragmatism*, p. 179.

al principles as syllogistic inference, it must be admitted that these latter also have some capacity for taking account of the differences of quantity and degree. All inference involves comparison; yet no comparison is possible save where some identical standard or unit is presupposed. But no sooner are the questions asked, "What is the extent of this identity?" "How much in common?" or "How far different?" than the aspect of magnitude becomes explicit. In other words, all qualitative comparisons seem ultimately to involve considerations of quantity. Even in expressions of difference in degree in regard to a single quality (such as "sweeter," "brighter," "louder," or "later"), the assumption of a common denominator or unit is clearly indicated with respect to which the series of degrees is constructed according to a definite decrease or increase in amount. Thus, if you say "This star is brighter than that," you plainly presuppose a standard (or unit of magnitude) of brightness by reference to which you define this qualitative experience. You mean that "this star is *as bright as* that one"; but that, in addition, "there is *some brightness in this one*

that is not in that one": in other words, *A* is as bright as *B* plus some more (how much, is left indefinite).[13] Similarly, if you say that a certain event (*B*) occurred later than another event (*A*), your comparison indicates reference to time conceived as a line or serial order. To begin with, you establish *A's* position on the line as your point of reference, and then find *B's* relative to it. Then, in locating *B* to the right of *A* in the linear temporal series, you become aware that relative to the whole of the preceding series taken quantitatively, there is *more* of the temporal line up to *B* than there is up to *A*. In other words, some of the extension up to *B* is not *A*, which is what you mean by saying the one is "later" (longer) than the other. Fundamentally, the comparison has to do with quantities rather than with relative positions in a serial order, it appears, and with the cardinal rather than the ordinal aspect of the series. For that matter, transitive asymmetrical relations seem peculiarly adapted to the expression of magnitude. And, although they spring from the same roots as the relations of the ordinary syllogism,

[13] That is, all *A* is *B* and some *B* is not *A*.

if we are not mistaken, in them the vague rudimentary marks of quantity utilized in traditional general logic are often supplanted by a developed mathematical symbolism, and thought is liberated to an unprecedented degree.

But how, on the foregoing account, it may be asked, is one to find a place for "intransitive relations" like "father of" or "one-inch-taller"? Such connections, if they hold between *A* and *B*, and *B* and *C*, cannot by definition hold between *A* and *C*. There can be no elision, no mediation through skipped intermediaries here; since, if *A* is an inch shorter than *B*, and *B* is an inch shorter than *C*, it is plain that *A* cannot possibly have the same connection with *C*. Accordingly, the assimilation of this field of intransitive asymmetricals to that of the traditional syllogistic type would at first sight seem to be impossible.

But here again the comparison of examples may prove worth while. Only by some such means does it seem possible to realize clearly how slight the difference between the ordinary transitive syllogistic and the so-called intransi-

tive relations may often be; a difference turning entirely, it would seem, upon the use of more specific terms of quantity. Thus, in the common *AAA* syllogism, our symbolism permits us to indicate only that:

All *A* is *B* [and some (not how much!) *B* is not *A*]

All *B* is *C* [and some (not how much!) *C* is not *B*]

Hence all *A* is *C* (and some *C* is not *A*); . . . this latter being equivalent to the "Some *B* that is not *A*," and the "Some *C* that is not *B*" taken together.

That the conclusion may here be taken to express the summation of differences between the first and second and the second and third terms is implied, I take it, in the principle of the *dictum de omni et nullo*, since "whatever is a part of a part is a part of the whole." In this syllogistic notion of inclusion, of container and contained, we have a quantitative process which apparently goes as far as the computation that *A* is less than *C* by as much as the sum of the differences between *A* and *B*, and *B* and *C*. True, where the measurement is carried beyond this in exactness, it is generally called mathematical. At the same time, it is fairly evident that the structural basis of both processes is the same.

Thus, the only important difference between the above and the inference "*A* is one inch shorter than *B*, and *B* is one inch shorter than *C*; therefore *A* is two inches shorter than *C*" is the introduction of the numerical unit in place of "some." In this case, the argument may perfectly well be restated:

All *A* is *B* and some (viz., 1 inch) of *B* is not *A*

All *B* is *C* and some (1 inch) of *C* is not *B*

Hence all *A* is *C* and some (i.e., summing the differences in extension of classes—2 inches) of *C* is not *A*.

Just as the relation is admittedly transitive where the mediating phrase is merely "shorter than" and the summation of differences stated in vaguer quantitative terms, so, substituting the arithmetical unit "one inch shorter," we must assume that the reasoning proceeds similarly through a common character by the same sort of transitive operation. Presumably, we may conclude that any relation, in so far as it relates, must be transitive, and that contrary appearances are due only to the more highly developed use of quantity.

Our purpose in the foregoing discussion has, then, not been to deny that the so-called non-

syllogistic arguments dealing with magnitudes and serial-order mark an advance in definiteness and precision over ordinary syllogistic reasoning, but only to protest against the attempted severance of the one from the other. In actual reflection, as it appears to us, no such dichotomy or mutual exclusiveness is present. But symmetrical and asymmetrical, transitive and intransitive aspects are constantly found in the same relation.

When the attempt was made to lay hold of the peculiar character of relations that distinguishes them from everything else, it was discovered that their essential nature lay in their being necessary connections. For relations that relate must have in them some inherent coerciveness that enables them to bind terms together; an efficacy moreover, that cannot be located in some irrational activity or in factual hooks and hinges, but must subsist in some systematic process arising among the terms themselves. A relation, in brief, can only mean a kind of autonomous structure that unites the elements of experience in an integral way. All relations are, furthermore, inferential; that is,

they are logical operations, not substantive couplings; and knowledge of them requires thinking them through, and not merely their acceptance or rejection as a factual presentation by the mind. Wherever any terms are linked together, an inference may be assumed to be involved. But this discovery does not point to the simple conclusion that *Alles ist Beziehung*. Such a result would be, in effect, as substantialistic as any that we have sought to avoid. After all, it is not the substance of things that matters so much as their organization and form.

CHAPTER VI

THE CONCEPT OF NATURE

I: A Schematic Picture

HITHERTO we have been concerned, for the most part, with the broad generic features of experience, with what is identical and recurrent in its structure. In addition to these universal aspects, however, there is a complex and practically far more insistent realm characterized by specific laws and phenomena, which is generally known as Nature. To a consideration of the latter we shall turn in the present chapter.

If, in order to clarify our minds as to the bearing of the different regions of experience upon each other, we attempt a feat of philosophic abstraction, and try to dismiss from the universe everything that can be done away with short of total annihilation, it seems not impossible that in the process we shall finally come upon an irreducible minimum of what is most essential to our world. For, when the experiment is tried —that is, when the glories and sorrows of our globe are progressively blotted out, and all the

tastes, colors, odors, that lend it pungency, are effaced, leaving only inaudible, invisible masses in motion, which in turn dissolve under critical inspection into the thin air of their mathematical properties—when all this is done, there will still remain, as appears to us, a framework of logical relations that serves as the ground-plan of the whole.

But, as the good anatomist dissects his specimen only to reconstruct it, so must we too endeavor, after analyzing our world, to build it up again quality by quality. Here, of course, we shall not take our handiwork very seriously. Enough that it is the fashion of the day to construct such tentative diagrams and cosmic hypotheses. On the whole, we shall be content if our schematic picture holds good in one respect; namely, in its depiction of the primacy of form over matter: in other words, in emphasizing the dependency of the rich and relatively "given" realm of nature upon the field of logical and mathematical relationships.

To this end we may portray the world as a pyramid or tier-like structure in which the different sciences or disciplines of knowledge rep-

resent the various levels of reality. Of course, such a schema is not intended to indicate any process of temporal evolution, or of degrees of reality, or progress toward perfection. On the contrary, any significance it may have rests entirely, in our opinion, in certain broad suggestions as to the order of logical dependence subsisting within the structure of knowledge and its objects. For, taking as our justification the fact that it is often impossible to separate the object of knowledge from the procedure of knowing, we shall allow a classification of the sciences to stand in our figure for the fields which they respectively envisage. Thus, at the base of our pyramid we may place logic and mathematics, or the disciplines especially devoted to the study of rational form. Above these appear the "natural" or physical sciences in ascending order from the inorganic to the organic, with the so-called sciences of value (aesthetic, ethics, religion) near the apex.

At least such a pyramid will have the value of illustrating graphically the dependence of nature upon the realm of mathematical and logical relations. By this is meant that, while the

former realm obeys and is determined by the laws of the latter (as well as by its own laws), the latter are not required to obey the laws of the former. Thus, if a man (i.e., a living being) falls out of a window, he must inevitably conform to certain laws of physics, mathematics, and logic (as well as to those of biology); but if, on the other hand, a brick (i.e., a merely physical and mathematical object) falls out, no one expects it to conform to biological laws. In this sense, there appears to be an irreversibility of direction, a one-way order about the strata of reality, which suggests the operation of some non-temporal process or logical evolution, and deserves close attention.

To our statements here some critics will undoubtedly take exception. "In maintaining that the so-called 'higher strata' depend upon the 'lower,' " they will say, "you overlook the important correlative that the 'lower' depend no less upon the 'higher.' Although it may be granted that the laws and phenomena of biology presuppose the validity of certain laws of physics, mathematics, and logic, it is false to assume that the processes of physics, mathematics, and

logic do not equally presuppose the former. There could, in fact, be no logical or mathematical relations were there no psycho-physical organisms of uniform behavior to think them out and formulate them. Since, therefore, no scientific laws subsist that are unassociated with psychological and biological phenomena (i.e., with the consciousness of human beings), the thesis is clearly proved that the lower levels of the pyramid depend upon the higher quite as much as the higher upon the lower."

To this objection a sufficient reply is to be found, as it appears to us, in a strong insistence upon the distinction between logical dependence and factual inseparability, and the refusal to admit that the one, which holds in the first case, is in any sense equivalent to the second, which holds in the other. For instance, everyone agrees, we may assume, that the Mendelian law of segregation could not possibly hold in its present form unless hybrids were hybrids, unless the mathematical ratio of 1:2:1 meant precisely what it does in our number system, or unless the units contributed by the parents to the germ cells of their offspring followed

certain atomistic principles of external relation,[1] in short, unless certain laws regarding identity, number, and mechanism were valid. On the other hand, it seems no less evident that the principles of identity, number, and mechanism do not, as we understand them, invoke or appeal to biological laws to establish their reliability. While an organism with a brain that functions in definite ways is undeniably required for *thinking* these mathematical laws, the integrity of the laws themselves is in no whit affected or validated by the accompanying biological and psychological phenomena. Of course, wherever knowledge takes place, a variety of more or less constant accompaniments is to be expected: spatio-temporal factors, oxygen, heat, movement, consciousness. Yet such factual concomitants are hardly to be mistaken for sufficient conditions of the process. For the latter, we must turn to the very different way in which mathematical and logical relationships underlie the biological world; here it is not

[1] Morgan, T. H., *et al., Mechanism of Mendelian Heredity,* p. 1. "The units contributed by each parent separate in the germ cells of the offspring without having had any influence on each other."

merely that without the former the latter could not *exist;* but that, without logical criteria to substantiate them, biological laws could claim no credible meaning or truth whatsoever.

Here, once more, we are urging the priority of reason over nature, arguing that the logical order must be assumed as ground of the temporal if any coherent account of knowledge and its object is to be given. In this respect, our view differs plainly from current theories of "emergence," which express a naturalistic tendency in so far as they call forth both the physical and the mental worlds from a basic, all-pervasive space-time. Furthermore, although the evidence forces us to concede that the so-called higher levels have something more and different in them than the lower ones, we find it impossible to admit, contrary to the emergent evolutionists, an element of unpredictable novelty, of that which is mathematically incalculable and saltatory in the process. According to their theory, wherever the combination of elements in a given manner gives rise to a resultant with properties distinct from those possessed by the constituents taken separately or in juxtaposi-

tion (as where nitrogen and hydrogen give rise to ammonia with its peculiar pungent smell, or where immensely complex combinations of carbon, hydrogen, oxygen, etc., produce protoplasm and the unique reactions of living tissues)—in all such cases, we must recognize something absolutely new and given, a creative synthesis of the emergent type.

But to this we cannot agree. Having depicted mind as wholly the product of nature and accepted other naturalistic premises, these thinkers, as it seems to us, wish nevertheless to escape the naturalistic conclusion. This they seek to do by stressing discontinuities everywhere. The phenomenon of chemical combination (as distinguished from that of mixture) furnishes them a favorite example of what they mean by emergence. In combination, as is well known, a complex whole arises possessed of characteristic properties quite different from those of the elements composing it. Now, according to the emergent theory, the nature of these new properties is not only different but absolutely *unpredictable* from those of the separate constituents, despite the fact that the com-

bination itself is completely determined by them.

But to accept determinism while rejecting predictability is, in our opinion, an error. Two such correlatives are no more separable than are the laws of ground and consequent. If the past is explicable in its essence, the future must be likewise. Furthermore, the development of atomic theory seems to allow for other interpretations of chemical combination than the emergent. Thus, there is said to be no actual coalescence or fusion of the atoms in combination, but only a regrouping of them in a definite pattern, the specific changes being due to the transfer of electrical charges among the constituents. Now, if all material substances are composed of such charges, it may well be that all sorts of sudden qualitative changes can be quantitatively explained in the future, in terms of the alterations in number and arrangement of these electrical units.

Lastly, although it may be admitted at higher levels that the characters of living organisms are different from those of their elements, just as the tonal quality of a musical chord is more than

the additive resultant of the component notes, this is no reason for accepting the "leap" without explanation by an act of "natural piety" as something incomprehensively "given." For, if the passage from one level of nature to the next is by logical saltation, as these thinkers seem to believe, our efforts at scientific deduction and analysis can have only a very limited application: an application restricted, indeed, to the properties of aggregates of the same order. Yet, if this be so, and if the appearance of syntheses and nodal points in evolution forces thought to admit discontinuities, abrogation of the law of ground and consequent, and the occurrence of events which it is theoretically impossible to deduce from the disposition of their components, then indeed, as it seems to us, the back-door is opened once more for the recall of the occult qualities. Under the new names of "emergence" and "creative synthesis," the old concept of creation (the advent of something from nothing and of the mathematically unpredictable) is introduced once more as certainly as in the *lux fiat* of Genesis.

Though there is much to sympathize with in

the movement, still the error must not be allowed to go unchecked that implies that synthetic results are incompatible with mathematical calculation and the prediction of consequences. Granting that the higher strata of the natural hierarchy, besides obeying the laws of the lower, obey certain laws of their own (that, as we said, a man who falls out of the window conforms not only to the law of gravitation but betrays certain distinctively vital phenomena), we may ask whether these laws which emerge as peculiar to certain strata may not be accounted as in the nature of specializations or complex adumbrations of the same principles that are operative in simpler form at lower levels. Certainly the attempts of the various sciences to correlate their phenomena with one another in so far as possible—of psychology to derive its concepts from physiology, of biology to comprehend its data in chemical and physical terms—seem to point in this direction. And, though such deduction enjoys only a limited success at the present time, and though the structure and properties of each particular field have largely to be discovered independently of

one another, still, as it seems to us, their apparent incommensurability may be explained more satisfactorily as the result of complicated action and the rare disposition of events under one set of principles, than as so many creative "leaps" or unaccountable discontinuities in the behavior of reality. A defense of logical mechanism in this sense, however, may be left to the next section.

II: The Irreversibility of Natural Processes

But how, someone will ask, can it ever be possible on a mechanistic hypothesis to explain the irreversibility of direction of natural processes? How account for the irrevocable sequences displayed in such phenomena as growth, age, and the degradation of energy? Surely, irreversibility is essential to the very meaning of evolution; indeed, it would seem to be the central fact which any pyramidal scheme must represent. How otherwise than by the creation of something absolutely, incommensurably new can multitudes of novel properties and structures make their appearance?

To answer, as we must, that the prodigious variety of nature springs from the various arrangements of a small group of elements among which the number of permutations and combinations is very great, may seem at first sight to be choosing the weaker side of the argument. Nevertheless, it is the side of computation and prediction as opposed to that of the incalculable and unpredictable. Furthermore, when one considers the endlessly fertile results already achieved through the manifold dispositions of a few units—the derivation of all the words in the dictionary from twenty-six letters, or the analysis of thousands of chemical compounds into some fourscore elements—one can hardly retain any great fear of restriction in a "closed" mechanical universe or apprehend that its possibilities will ever actually be exhausted. Not only does such a view allow for an unimaginable richness and complexity of events, all of which are, theoretically at least, deducible from their antecedents; but, in addition, it offers an explanation of the non-recurrence or irreversibility of natural processes, which is in accordance with the theory of probability.

Wherever the disposition of events leads man (as it does in most accounts of evolution) to describe phenomena in terms of some inherent progression, or tendency, or historical "force" having direction, there is something present, as it seems to us, that can be statistically and mathematically explained. In the words of Royce, "the term *tendency* is, in every exact usage which you can give it, an essentially statistical term. To say that *a* has a tendency to lead to *b* is to declare that a more or less certainly and definitely known proportion of events of the class *a* are followed by events of the class *b*."[2] Accordingly, where one-way movements appear in nature, it would seem that statistical methods are to be looked to as the key to their solution. And, of course, the irreversibility of physical states and the continual emergence of unique phenomena offer striking examples of such tendencies. But when, as we shall see, nature's disinclination to repeat herself is regarded from a statistical point of view, it loses altogether the character of an inherent impossibility and comes to signify only the extreme

[2] *Science*, vol. XXXIX, p. 559.

mathematical improbability that a complex system should pass twice through exactly the same state. While, admittedly, such an hypothesis as regards the world-process is not open to confirmation in detail, its appeal to the mechanism of mathematics, together with certain facts in its favor, make rather plausibly for its acceptance.

Let us try to make clear by a simple illustration how the phenomenon of "creative novelty" (or unrepeatability) may be accounted for without recourse to mysterious leaps and discontinuities. Our example, let us add, is adapted from the work of M. Guye on *Physico-Chemical Evolution,*[3] in which he offers an interesting theory of evolution based upon the calculus of probabilities.

It is a matter of common observation that when two powders, or even two liquids or gases of the right sort, are shaken up together in a receptacle, a very nearly uniform mixture is obtained within a short time. Suppose we try the experiment with ten grains of white and ten grains of black powder, exactly like each other

[3] *Physico-Chemical Evolution* (Tr. by J. R. Clarke), p. 49f.

in size, mass, and all save color. At the beginning, these grains are placed in a narrow tube with a bulb at one end: the ten white grains one above the other in the upper part of the tube and the ten black grains similarly in the lower part. Next, suppose we reverse the tube, shake it violently, and return it to its original position. Following the agitation we shall probably find the grains arranged differently, so that now the powder appears gray throughout to ordinary observation, whereas, before, half looked white and half black. Indeed, no matter how long we continue shaking the contents, the powder will, in all likelihood, persist in its appearance as a gray mixture. How is this to be explained?

Apparently, the simplest way of accounting for the tendency of the powder to remain a more or less uniform gray, and its failure to return to its original unmixed state, is by reference to the theory of probabilities. According to the formula n(n—1) (n—2) . . . 3.2.1, the permutations upon 20 entities are 2,432,902,008,176,640,000; which means that the twenty grains of powder can be arranged in

the tube in over two million million million different ways. Of course, among this host of permutations there are a certain number which, although not precisely the same, nevertheless produce indistinguishable impressions upon our eyes; for, since the grains differ only in color, we should not be able to differentiate within the group of ten of the same color. Accordingly, the number of permutations that we can really discriminate is somewhat less; in fact, 184,756. This result is, of course, derived by means of the formula

$$\frac{n(n-1)(n-2)\ \ldots\ (n-m+1)}{1.2.3\ \ldots\ m}$$

where n is 20 and m is 10. Thus, the chance that, at any given agitation of the tube, the powders will separate into the original black and white groups is 1/184,756; whereas the chance of their returning to exactly their initial state is 1/2,432,902,008,176,640,000. Our conclusion in regard to the experiment is that the reason that the powder does not return to its original unmixed state is not that such an event is intrinsically incapable of occurring, but only that the chance of its happening is so slight

as to be practically negligible: the result is virtual irreversibility.

Obviously, this type of explanation may be extended to all sorts of natural processes that involve direction. Moreover, the prevalence of atomic conceptions regarding nature at the present time makes its application easier. That is, situations that imply the granular structure of matter or biological units like chromosomes should presumably lend themselves to statistical computation. The complexity, however, of the actual data is a prodigious drawback. Whereas the illustration above dealt only with a few grains of powder, most natural phenomena, on the contrary, comprise far larger groups of units; and, in consequence, the number of possible dispositions involved becomes incomparably greater.[4] Nevertheless, despite the fact that the intricacy of nature must probably forever baffle us in regard to matters of application and detail, it seems as

[4] *Ibid.*, p. 56 note. "As one cubic centimeter of gas at 0° and 760 mm. contains 3×10^{19} molecules, the number of permutations which can be made with the same number of grains would be 3×10^{19}."

if we had here at last a clue that might explain the apparent asymmetricality and perpetual novelty of phenomena.

The second law of thermodynamics, for instance, which tells us that energy tends to be dissipated in the form of heat and that the quantity available for mechanical work is continually diminishing, may now be regarded as a general expression of the extremely slight chance that a complex system will return to its previous state. That molar motion tends to be converted into molecular motion (and what we call heat) seems due to the fact that the *collective velocity* of the molecules of a body moving a certain way under the action of its weight or an external force, is gradually transformed (through friction, collisions, and other causes) into an *uncoördinated agitation* of the individual molecules—an internal motion which tends to be approximately equal in all directions. But once this symmetrical, distributed motion of the parts of the system is realized, it becomes highly improbable by the laws of chance that these independent agitations will ever unite again in a concerted, asymmetrical motion of the whole

system in a given direction. Thus, what has hitherto appeared as absolute irreversibility and has been explained *(ignotum per ignotius)* as somehow due to the asymmetrical nature of time, which rendered the opposite order of succession inconceivable, turns out to be only the statement of the logical improbability of an event, extremely unlikely but by no means impossible.

Recurring once more to the example of the powders, a point to be noted is that, although the configuration that would separate the black and white grains into two groups is neither more nor less probable than any other, it would nevertheless, if it occurred, confer upon the system *decidedly different properties* from those possessed by the great mass of configurations. For, whereas the vast majority of arrangements would appear as approximately uniform gray mixtures throughout, the occurrence of the unmixed white and black groups would offer a striking variant by contrast. Furthermore, once in a long, long while such an extreme variation, quite unlike the "run" of the others, is to be expected. When such a rare type of fluctua-

tion actually appears, it will be hailed as something absolutely new, as a startling innovation. As a matter of fact, however, it is not "new" in any sense of being mathematically incalculable or of resting on a different footing from other possible permutations, but only in that it represents a large degree of divergence from the mode or mean.

Perhaps even from so simple an illustration, we may catch a glimpse of the way in which the so-called higher orders and levels of our world may have come about. Life itself and other important emergents may perhaps plausibly be regarded as the result of extraordinary dispositions of chance. It is at least conceivable that in the course of age-long processes of association and dissociation of the elementary energies of our universe, a vast number of possible arrangements should have been realized and that, among these, the peculiar combinations of carbon, hydrogen, oxygen, and other elements that furnish the basis of living matter should have occurred as fluctuations of a very rare type.

III: Is Nature Definable in Terms of Sense-Perception?

But these highly speculative conjectures regarding irreversibility and the appearance of new levels have already diverted us too long from our main question which was to discover, if possible, the essential qualities that distinguish the world of nature or existence. While we have already conceded the claims of the predicates *is* and *is true* to divide the universe between them, little room is left for doubt as to which of these is here regarded as the more fundamental. This opinion is not to be taken, however, as in any way prejudicial to, or a denial of, the rights of existence. By the world of existence, I suppose we mean broadly the same thing as nature: a realm inclusive of all feelings, sensations, qualities, things, and events —of everything, in short, save formal patterns and logical relationships.

Very often this region is defined ultimately in terms of percepts. What has existence is identified with what is revealed to the senses; while by nature as a whole is meant the totality of what can possibly be so observed. Thus, in the

words of Leibniz' famous postulate of observability, we may assume that "nothing fundamentally outside the sphere of observation can have being in the physical sense"; or, in the language of a more clear-cut empiricism, "to be is to be capable of being perceived."

Anyone who examines the evidence that has led so many thinkers to the phenomenalistic view of nature must acknowledge that its bulk is impressive. Not only does our knowledge of the surrounding world begin with sense-impressions; but, through these contacts, we seem to make the acquaintance of something absolutely real and veridical. Moreover, when we seek to define natural objects, the list of their properties (mechanical, chemical, or aesthetic) seems ultimately derived from perception; while, if we appeal to our best attested modes of gaining knowledge (such as occur in the natural sciences), it is found that observation and experiment with sensible instruments are there the accepted method of investigation. Indeed, only by narrowing the sphere of existence to the strictly observable does it seem possible to close the door on the immeasurable forces and

occult qualities that are constantly pressing for recognition as part of nature. The conclusion of empiricism, in the words of Mill, is therefore that "All we know of objects is the sensations which they give us, and the order of the occurrence of those sensations."[5]

But, however this account may serve as a description of immediate experience, its failure must be admitted, as it seems to us, to explain the apparent objective status of the external order; for, in the first place, the mere insistency of a sensuous presentation is insufficient to explain the existence of an object, whether the object be merely a sensation or a substance underlying it. True, the given cannot be questioned for the moment, because it overcomes the organism. Nevertheless, mere organic shock is not to be mistaken for proof. Our acceptance of a blow on the head or a brick on the toe does not establish the existence of any external object; for a blow is not knowledge, and therefore can neither support nor refute any theory as to the nature of things, Dr. Johnson to the contrary. What the organism cannot assimilate or dodge

[5] Mill, J. S., *System of Logic*, bk. I, ch. 3, sect. 7.

in the environment, it accepts because it has to; but this acceptance (called "adaptation") is only equivalent to incapacity for resistance, which is something very different from intellectual comprehension and assent. In brief, the trouble with so-called cognition based on pure sense-experience is that, so long as it remains a matter of bare perception, expectation, and recall, it can neither prove nor disprove anything, being merely animal faith, and, as such, without value as knowledge; whereas, on the other hand, in so far as it ventures beyond the dumb deliverances of sensation, it has shifted its ground and is claiming an authority for which mere sense-experience is no sufficient warrant.

A further objection to the perceptual view of nature is to be found in the conflict which arises between the unshared, private character of sense-experience and the shareable, public constitution of the physical world as it is generally conceived to be. Whereas nature itself is ordinarily supposed to enjoy an objective status common to all observers, sense-impressions are thought of as functions of the individual organism and its peculiarities. Because they

are open to the inspection of only one observer, it is admittedly difficult to establish agreement in regard to them.

An attempt to resolve this antinomy is being made at present through a rigid insistence upon the applicability of the uniformity of nature to both classes of phenomena. In actual import, it is pointed out, uniformity raises no distinctions between the psychical and the physical. Rather, its value consists in postulating the unity and constancy of structure of our world. Whatever is, uniformity seems to say, is such that it could be duplicated by somebody or something else under the same conditions. In this way, the public and private aspects of things are held to be reconciled through the assumption of the lawfulness of nature. Thus, although in describing a table, each observer observes its shape, size, and color differently, depending upon his sense-organs, his place in the room, the time of day, and so on; nevertheless it is conceivably possible to correlate these different perspectives with one another, to reduce them to a common scale, so that a definition may finally be obtained of what the object may be ex-

perienced as from different points of view, which may be said to constitute the real nature of the table.

Now the main point in such an account of the physical object is that it becomes ideally possible for any observer to change places with, or be substituted for, any other observer, so that, under the same conditions, the same experience could be duplicated by somebody else. With this principle we are inclined to agree; the more so since, so far as we can see, it resolves the dualism between the perceptual and the cognitive sides of nature entirely by reducing the former to terms of the latter; for the postulate of ideal representation or substitutability clearly denies any element of exclusive particularity in things—a denial which is apparently born of a reliance upon abstract conception and universals rather than upon concrete experience. From the point of view of a theoretic science like geometry, for instance, superposition is a familiar operation, and any solid can be made to coincide exactly with another like it; but, from the standpoint of sensuous experience, the coincidence of solids is an actual impossibility, some-

thing that simply cannot be produced. Faced here with an alternative, the principle of uniformity seemingly urges us to prefer theory to fact. After all, we may say to ourselves, the difficulty of making two bodies occupy the same place is a purely practical one, a defect in our present technique of handling solids. There is no reason to suppose that there is anything inimitable, unshareable, or unique in the constitution of each separate object. Such old-fashioned notions as that two things cannot be alike in nature, but that each individual makes a distinct, ineffable contribution to the world which cannot be duplicated or repeated, such ideas must be discarded. The entire approach to nature through methods of measurement, has, indeed, to rule out uniqueness from objects (or to treat it as a fugitive limit which vanishes where knowledge is perfected) in so far as observations are related to one another and assimilated to a common scale. In brief, from the standpoint of uniformity there can be nothing unshareable and incapable of substitution for the enlightened understanding where the latter is properly qualified and conditioned.

Lastly, where perceivability is made the definitive mark of nature, it seems fairly clear that reference to an experiencing organism or organisms, to some observer (real or hypothetical), has to be included as an essential part of the objective world. At any rate, all the experiences and perceptions that we know anything about involve reference to a responsive organism no less than to the stimulus of an environing situation. Yet the task of discovering such an observer or observers capable of acting as the source of all responses is anything but easy. Within the limits of actual experience, it is generally admitted, no synthetic mechanism adequate to the requirements is to be found. And even if there should be, hardly anyone would be willing to limit natural processes to those revealed merely through the present perceptions of living organisms. At the least, reality would seem to include, in addition, objects of future, past, or ideally possible experience. In any satisfactory theory of nature, allowance must be made for extensions beyond the empirical present. It is not surprising, therefore, that in the three variants of the perceptual view

popular in current thought, chief emphasis is laid upon the potential and unrealized aspects of experience. In the following section, a brief review will be offered of these several conceptions, showing how observability, imaginability, and concrete intuition are utilized by them as ultimate criteria of the physical, and how the facts of nature are interpreted as hypotheses, fictions, and regulative ideas respectively.

IV: Hypotheses, Fictions, and Heuristic Principles

When a radical empiricist faces the question "What is the world made of?" his first reply is usually in terms of sense-impressions or given presentations. Yet this simple answer has generally to be modified in the face of the complicated structures of science and common sense —worlds crowded with things, selves, events, laws, and substances. At first glance, at any rate, these latter appear to be much more than sense-qualities, since they seem to have a continuous existence, organization, and causal efficacy which set them apart from the flux of momentary sensations. Given sufficient time,

however, the phenomenalist is usually able to explain these also to his own satisfaction as extensions of the realm of sensa. "Things and selves," he will probably say, "are only complexes of sensations projected by us into an independent world, and endowed by us with causal efficacy and permanence in order to lend some measure of unity and constancy to the stream of impressions." Similarly, laws and events, matter and energy, are simply so many hypotheses or conjectural links that arise as constructions of the scientific imagination. Just as a chair is comprehended by the phenomenalist as a suppositious entity derived from patches of sense-data pieced out by associative inference, so likewise the theories of science are regarded by him as so many conceptual schema based upon certain observed correlations which have been raised by imagination to the status of supposedly objective reality. Yet, at bottom, both worlds are no more than a bombardment of sense-impressions or a series of twinges aroused in the organism. To be sure, the sensa have undergone a process of sorting-out and have been organized into more or less perman-

ent constellations; still, in their essence, all objects, phenomenalistically conceived, are mere eddies in the stream of sensation.

Common sense and science are admittedly filled with concepts that are not literal descriptions of actual sense-experience, yet which are generally believed to correspond to what is real. In many cases, they have been confirmed for all practical purposes through a large amount of consilient evidence accruing to them from different fields. Again, the existence of entities quite remote from ordinary observation is often established by means of the principle of continuity. Indeed, this latter law has over and again (notably in the applications of the telescope and microscope) vindicated man's right to extend the same principles with which he operates in unaided immediate experience to regions beyond direct experience. In notions like those of the ether, the electron, vitamins, etc., we have entities that, though remaining directly intangible and invisible, appear none the less to have their existence satisfactorily attested through certain indirect, sensible effects. Such objects, although we can-

not incontestably prove their reality, are nevertheless assumed to exist for the reason that they seem to be continuous with, and required by, the immediate conditions of our actual experience. In them, we have a class of hypothetical or "possible" objects of experience which are affirmed by virtue of their similarity to, and congruence with, actual perception. Assuming that various limitations of our organisms were removed and our sensible powers vastly magnified, multitudes of objects that at present can be only deviously inferred, would presumably swim into the range of immediate perception.

In conclusion, what distinguishes these hypotheses from other objects of imagination, such as fictions and regulative ideas, is the fact that they always claim to represent reality. Belief in a genuine, objective state of affairs, with which the imaginary theory or item is in correspondence, is essential. With the accumulation of further evidence, or with the achievement of some more properly circumstanced perception, it is hoped that the hypothesis shall be proved true; that is, that it shall be found to coincide with a real, external situation.

A fiction, on the other hand, is a provisional construct of the imagination, which does not claim to be a description of the objective world. Examples of such fictions, are familiar to all of us in the notions of perfect circles, absolutely straight lines, mathematical points, timeless instants, potential energy, curved space, *persona ficta* in law, and so on. Not only are such fanciful inventions said to be false to fact; they are often self-contradictory as well. Nevertheless, despite their inconsistency, many of them are held to have proved extremely fruitful in science and practical life; and, as such, to have justified themselves by virtue of the problems that they have helped to solve.

While our intention was to withhold criticism of these various views until all had been presented, a few words of comment here regarding fictionalism may not be amiss. The most obvious inquiry to raise is why unreal and untrue ideas should promote our knowledge of truth and reality. Why, for instance, should Euclidean geometry, which is that of a fictional space-system infinitely distant from a gravitational center, prove so fruitfully applicable to

bodies in strong gravitational fields? What value is there in pretending that reality is as it is not? Or why should contrary-to-fact statements about the world be precisely the ones to hold good of it? Questions like these in regard to fictionalism appear more or less unanswerable; while, where the attempt is made to extend fictionalism to the proportions of a universal view, it becomes plain that fictionalism can only be regarded as itself a fiction. In brief, knowledge on such a view becomes self-deception on a grand scale.

The third variant of the perceptual view of nature makes use of regulative ideas. As such, it may or may not accompany either or both of the others. Regulative ideas are described as principles of systematic unity which serve to organize thought and action around certain imaginary ideals of experience, despite the fact that such ideals suffer from inner contradiction. Aesthetics, ethics, religion, and even natural science are said to utilize them; while familiar examples are cited in our notions of totality, perfection, beauty, freedom, and immortality. Though perceptually unreal, such objects are

regarded *as if* real; and, though full of antinomies, remain the pole-stars of life and thought. In them is embodied our desire of the impossible. When Kant spoke of the idea as a *"focus imaginarius,"* it is said that he clearly recognized the imaginary, as opposed to the intellectual, character of the ideal. In any case, identification of the absolute with the imaginary betrays a marked affinity with modern analytic psychology, where ideals are accounted as "unfulfilled wishes," as what we want and cannot get, imaginatively projected into a realm of compensatory fictions regarded *as if* real.

Regulative notions are held to resemble fictions and to differ from hypotheses, in that they lay no claim to correspondence with objective reality. As "flying perfects," ideals are in their nature unattainable. Yet, at the same time, they differ from ordinary fictions, in that they cannot be adopted, created, or discarded entirely at will. The fact is, apparently, that some principle of systematic unity is necessary as a guiding concept in every theoretical endeavor: thought cannot do without it. Furthermore, these same prescriptive notions of totality seem

no less indispensably binding in the realm of practice. Wherever action is fully conscious, there has to be some value on ahead, some goal or incentive to keep men striving; and even though we recognize that the goal is unobtainable in concrete actuality, we continue to press toward its fulfilment. Thus, in observing the function of the regulative ideas, the fundamental antinomy of sensible experience becomes visible: since, on the one hand, experience always seems to demand a concrete whole of immediately realized intuition; while, on the other, the state of actual living remains always fragmentary and incomplete, never yielding any such inclusive perceptual synthesis.

Here, in the end, lies the weakness of empiricism. At bottom, despite their dissimilarities, the foregoing views owe their rejection to a common fault; for, at the same time that all three define nature in the language of concrete perceptions, imagination, and intuition, they nevertheless describe it in terms contrary to actual, immediate experience. In other words, they conceive the physical world to be so and so observable, when it is *not* so observable. Where-

as the fictional view devotes itself outright to the creations of fancy, and while the regulative pursues the (perceptually) impossible, the believer in hypotheses takes refuge from all difficulties in his doctrine of "possible perceptions," defining transcendent objects as such that "If there were a perceiver, he would perceive thus and so in an imaginary case." But either this is a mere tautology, as we said earlier, asserting that "If things were not as they are, they would be as they are not," or else it contradicts the empirical principle by assuming experience to be experienced as it *is not* actually experienced.

In our opinion, doubt may well be expressed whether any view which holds that the reality of nature is communicated in sense-perceptions can convert these private feelings and observations into universal, objective characters of that world merely by translating them into the form of hypotheses or indispensable imaginaries. For the only perceptions that we know anything about, belong to actually existing somebodies. Yet no actual observer or observers could compass in imagination the infinity of aspects or even a fraction of the *totum simul* of intuitive

hypotheses necessary for a picture of nature. As a matter of fact, imagination, although it enjoys more or less indefinite powers of repetition, is always inevitably limited. Being essentially a faculty of direct presentation, it is circumscribed by the range of the organism's receptors and by its susceptibility to fatigue and exhaustion. Our present consciousness, for instance, could hardly picture, even in successive acts, the "look" of a trillion-sided figure, or represent in any concrete way the experience of a being with a hundred senses. Yet feats of imagination far transcending this would be required to encompass directly all of nature.

V: The Relational Character of Nature

In conclusion, since no individual organism or combination of organisms or observers can be conceived adequate to account for the presumed wealth of natural processes, it seems advisable to abandon the perceptual view of existence, and to assume instead that nature is to be finally tested by logical rather than by perceptual criteria; and, though we may still admit that reference to an observer is implied in every

scientific law or phenomenon, we may henceforth conceive this percipient as a purely logical subject or necessary condition of knowledge. The real import of such a subject, the presence of which is attested in and by the relativity of phenomena, is, as it seems to us, neither that of a hypothetical experience, a fiction, or a *focus imaginarius* but rather that of an epistemological postulate as to the cognizability of nature. After all, the intelligible character of events has to be assumed where knowledge is possible. Nevertheless, natural laws contain no reference to a focus of perceptions analogous to, or continuous with, our *de facto* powers, but only to mind as a universal or to the universe itself taken as a totality of relations. Indeed, it is extremely doubtful whether such a system of operations could, in any comprehensible sense, be called an "experience." Recognition of an inexhaustible network of relations, in which the law of same grounds, same consequents, holds, yet in which an indefinite number of particular terms could be substituted without upsetting the pattern, seems pretty much the substance of what we have learned about nature.

CHAPTER VII

NATURE AND PROBABILITY

I: The Meaning of Measurement

THE tendency of the modern mind to translate nature into so many forms of motion or energy is one way in which the growing persuasion of the relational character of things finds expression. In the light of recent developments, the significance of the long crusade in science and philosophy in favor of the primacy of the primary qualities becomes increasingly clear; for the whole movement toward the reduction of nature to its space-time aspects has led plainly in the direction of recognition of the mathematical, quantitative, and relational basis of existence. What has inevitably displaced the secondary qualities and immediate sense-data in the interpretation of nature, is the fact that the deliverances of the special senses are apparently in terms of a particular organism; whereas nature itself claims to have common, public, and shared attributes. In the case of the primary qualities, however, since more than one sense is generally involved,

comparison becomes important, with the result that individual peculiarities tend to be minimized. Take the awareness of motion as an example. Usually both visual and tactual sensations enter into it, thus requiring a judicious correction of the verdict of the one by the other. For, since no sense experiences another sense (that is, since touch cannot see or seeing feel), the task remains largely a matter for abstract conception. In the end, it would appear that escape from the privacy of sense perception and the unshareable intimacies of subjectivity is to be achieved, not through the translation of nature into the experiences of a hypothetical perceiver, but through the instrumentality of the primary qualities; since in them comparison, calculation, and relational factors play a chief part, and hence by their means, if any, communicable results would seem to be obtained and objects reduced to a common scale.

The meaning of nature from this standpoint is probably best reflected in Planck's phrase "What can be measured is what exists." This is not to say that only what is open to inspection by the senses exists; for much is generally in-

cluded in nature that cannot be so observed. For instance, things like quanta and electrons are both, so far, unperceived directly or sensuously (and perhaps will remain so); yet little doubt is felt of their reality because of the reckonable results that follow from them. Rather, the import of the statement is that whatever can be treated numerically or as subject to determinations of magnitude must have being. Science has capitalized this recognition of measurement as the criterion of existence in its quantitative method. For it, whatever can be translated into, or correlated with, the number and arrangements of certain homogeneous units has reality. Not only the size, figure, and movements of phenomena, but their sounds, colors, and temperatures are tested for objectivity by science in this way.

When we wish to measure anything, as we know, the first thing to do is to select (on the basis of chance or convenience) some *specimen* of the kind of thing we want to measure, and to adopt this as our standard or unit. Thereafter, the operation consists usually either in comparing examples of the thing to be meas-

ured with this typical specimen, or else in applying the unit itself again and again to the particular object to see how many times the one is contained as a multiple or fraction in the other. In this way, things are said to be tested by reference to a standard in terms of which they are defined. Thus, if two things are found to be equal to the unit or standard in a certain respect, they are equal to each other in the same respect; or, if one of them is estimated to be nine times the unit in size or weight, while the other is only three times the unit, we say that the one is three times as great in size or weight as the other. But, since the selection of the unit of measurement is more or less of an arbitrary matter, the question as to how many times it goes into the object, which is the specifically quantitative question, is answered differently according to the standard chosen: whether, in respect of length, for instance, it is a millimeter-stick, a yard-rule, or light-year. Nevertheless, despite this element of caprice in the selection, and despite the fact that objects are definable relative to a specific system of measurement, the relationship between the in-

strument, the observer, and the object is held to remain constant during the process; since, without the assumption of invariancy, measurement itself becomes impossible.

Here attention is called to the network of methodological postulates underlying mensuration. Unquestionably, the analysis of these, if it could be accomplished, would be of the utmost interest. For one thing, as we have said, we assume that our standards remain unchanged during the act of measurement. Although it seems possible that actual yard-sticks and spring-balances expand and contract during the process, because of the effects of temperature, friction, and so forth, and although we have no empirical guarantee that our organism and observations likewise may not undergo variations without our being able to detect them, we none the less stipulate that a yard remains a yard and a gram a gram both with respect to the object and to the observer during the course of experimentation. Even in so simple an experiment as that of constructing a circle with a bit of chalk and string, we posit that the string is exactly of a certain length, that its

dimensions remain constant throughout the operation, and that the relation of the circumference to the radius holds the same for all circles. And it is presumably precisely because these presuppositions of the process are taken as universals and not as particulars, as invariants and not as variants, that we can hope in measurement to obtain some genuine discovery as the result of our postulations, and to take out something other than we put in.

At this point, we touch upon one of the most difficult aspects of measurement, the way in which its empirical and logical aspects are reconciled with each other. Often, in weighing and estimating events in nature, it seems almost impossible to distinguish where the physical facts leave off and the deduction begins. Indeed, much that goes by the name of observation and experiment would appear to be little more than the attempt to learn from experience what was already implicitly presupposed in our methodological assumptions. As some one has said, not a few scientific investigations are like discovering that a yard has three feet by measuring a large number of particular yards. Re-

searches like those tending to establish the conservation of energy might seem to be a case in point; for presumably, the general principle has already to be accepted *a priori* as an essential condition of scientific measurement.[1] Hence, there may seem to some a touch of supererogation about the scientist's laborious efforts to verify it in different fields (in the transforma-

[1] Cf. Eddington, A. S., *The Nature of the Physical World,* pp. 236-239. "Conservation of energy, mass, momentum, [etc.] . . . are truisms." "Granting that the identification [of their subject-matter] is correct, these laws are mathematical identities. Violation of them is unthinkable." Such laws, Eddington goes on to say, are the scientist's "form of accounts" which, however, he has tended to accept as "part of the nature of things." The physicist has been like the ancient college Bursar who discovered that the great law controlling the college was that "For every item on the credit side an equal item appeared somewhere else on the debit side" of his ledger. "Once again," continues Eddington, "we find that a fundamental law of physics is no controlling law but a 'put-up job' as soon as we have ascertained the nature of that which is obeying it. We can measure certain forms of energy with a thermometer, momentum with a ballistic pendulum, stress with a manometer. Commonly, we picture these as separate physical entities whose behavior toward each other is controlled by a law. But now the theory is that the three instruments measure different but slightly overlapping aspects of a single physical condition, and a law connecting their measurements is of the same tautological type as a 'law' connecting measurements with a meter-rule and a foot-rule."

tion of mechanical work into heat or in the metabolism of the organism), quite as if he had not, in a manner, placed it there in his original assumptions.

Another postulate of measurement is presumably to the effect that a constant relationship holds between the points of view of different observers; so that, when their perceptions vary, they none the less vary in fixed proportion. For the fact that the measurements made by one observer can be translated into those of another, as they apparently can, is explicable only on the assumption of the subsistence of a set of invariant determinations underlying them. Thus, although what was a foot for one percipient might appear as a yard to another, the ratio between these magnitudes and the relative proportions of the observers' worlds would remain constant under both conditions of measurement. Just as it is possible for the same observer to measure a table in terms either of a foot-rule or of a yard-stick at his pleasure, without any change occurring in the relations of the object despite the change in measuring-scale, so, likewise, it is conceivable that the estimates

of organisms of different constitution and perspective can be successfully correlated with, or translated into, each other on the same supposition of relational constancy by which, wherever the standard of reference changes, everything alters in the same ratio. Apparently, it is only this assumed invariancy that renders it possible for each, after he has observed for himself under his own conditions, to compare his results with those of others observing under their respective conditions. Were this power of discerning likenesses and differences not finally genuine, and were there no discoverable fixed correlations or principles of transformation from one system of measurement to another, the substance of most of our thought about the world would presumably have to be repudiated.

This is only to say once more and in a slightly different way that, if there is such a thing as knowledge, the transcendence of the logical over the biological has unquestionably to be admitted. Although each observer appears to be confined, physically speaking, within a local environment and forced to view the world from his individual perspective, nevertheless, by means

of calculation and measurement (which are essentially acts of comparison and reasoning), he seems able to overstep the limitations of perception to a degree at least, and to lay hold of certain invariant dependencies of a cosmic, and not merely of an anthropocentric, character. If science is not to admit that it is itself (like its ancient enemies, animism and superstition) a mere technique of anthropomorphism, dedicated to the proposition "Man is the measure of all things" (emphasizing "man" and "measure"), it must recognize that thought and experimental methods grasp objective relations that transcend the peculiarities of particular organisms and that there are laws of nature which hold for observers in any frame of reference.

Let us not make the mistake of interpreting the general conception of relativity, which is permeating reflection upon scientific method at present, as simply the canonization of our biological peculiarities and of the observer's powerlessness to detach himself from the limiting conditions of his environment. For, although the theory does call attention to the impossibility of the observer's jumping out of his skin in a

physical sense, it nevertheless admits the feasibility of his doing so intellectually. Thus, despite the fact that we are confined within the barriers of our animal organism perceptually, we are none the less able to free ourselves from the encumbrance, in part at least, theoretically, by laying hold of certain mathematical invariants, and by combining the measurements of different systems with one another in such a way as to cancel out their individual peculiarities and to obtain descriptions of physical processes that hold for a plurality of systems. Here, as so often in knowledge, to take account of our limitations is to transcend them. In the end, what the principle of relativity has apparently to bring home to the world is not anthropomorphism or individualistic relativism as a theory of knowledge, but a stricter realization of the difficult requirements of true objectivity.

In brief, the competence of reflection to stand outside itself and to assess (after the fashion of a universal spectator) the relationships that hold between its myriad centres in the natural order, must be frankly recognized as an assumption justified both by results and by the incon-

ceivability of the opposite, and not be admitted (as it is now) covertly in the midst of denials of the very point at issue. For how, unless the self-transcendence of reason be allowed, can a circle be avoided in the technique of measurement itself, where so often the act and instrument of measurement are part of the system measured? How, indeed, when, in physics, laws are discovered by the aid of physical instruments which are themselves subject to the laws they seek to establish; and when the products of research in the historical sciences are themselves a part of history? Mr. P. W. Bridgman has recently dwelt upon some of the logical difficulties of this sort in physics in a passage which it may be well to quote:

> In order to ascribe any simple significance to postulates about the velocity of light, it would seem that we must have an instrument for measuring this velocity, and therefore for measuring time, which does not itself involve the properties of light. To do this we might seek to specify the measurement of time in purely mechanical terms, as for instance in terms of the vibration of a tuning fork, or the rotation of a flywheel. But here again we encounter great difficulties, because we recognize that the dimensions of our mechanical clock may change when it is set in

motion, and that the mass of its parts may also change. We want to use the clock as a physical instrument in determining the laws of mechanics, which of course are not determined until we can measure time, and we find that the laws of mechanics enter into the operation of the clock. . . . In practice, the difficulties of such a logical treatment are so great that the matter has been entirely glossed over.[2]

Escape from epistemological difficulties of this sort seems impossible, in our opinion, without frank recognition of the right of logical postulation and the competence of the intellect to stand outside different systems and to combine and cancel out, as it were, their individual peculiarities. Measurement, on this view, becomes a type of rational comparison.

In fact, the essence of measurement consists, apparently, in the comparison of two or more things with a third, this latter being taken as the unit or basis of the operation, by reference to which the others are judged "more" or "less" like the norm. When the comparison is satisfied with an answer simply in vague terms of "more" or "less," or with an arrangement of the graduated intensities of a quality (as when we

[2] Bridgman, P. W., *The Logic of Modern Physics*, pp. 70-71.

classify stars according to their degree of brightness), in such cases the process of estimation is called one of intensive magnitude. But when we go on to ask definitely "How much?" we pass into the sphere of extensive magnitude, where mathematics is the usual instrument and in which the unit admits of being equated with, or contained in, the object an exact number of times. (In cases where a remainder is left over, the problem is solved by the use of a smaller subdivision or supplementary unit.) The three best known instruments for such measurement are presumably the centimeter-stick for estimating length, the balance for weight, and the clock for time; with their three corresponding units, the centimeter, gram, and second, expressed in the so-called C.G.S. system.

In no type of quantitative estimation, in our opinion, is any qualitative resemblance between the object and instrument essentially required. Accidentally, of course, such resemblances are often present. But all that is really necessary is the presence of a constant relationship between the two which, as in the case of the concomitant variation of mercury and temperature, serves as

the common denominator of the quality measured. Nor is the notion of superposition, borrowed from geometrical method, essential to it. Although, in estimating lengths of time and space, the notion of an ideal coincidence of unit and expanse may seem plausible, the same idea cannot be applied to weight, where the operation is rather one of counterpoise or balance. In weighing, indeed, the very nerve of the process apparently becomes clear, which is that of equivalence of the capacity of mutual replacement, sameness of quantity rather than superposition in a spatial sense.

II: Nature as the Reservoir of Magnitudes

Before proceeding farther, let us pause to cast a retrospective glance at our position. The realm of being or nature has been broadly distinguished in the foregoing pages as the object or datum to which logical relations and determinate principles apply. Whereas form is and remains the *regula* of our world, nature is the *regulatum* that obeys it. But, if it be objected that this notion is too wide, comprising as it does the total catalogue of conceivable objects (so

that "whatever is, is natural"), nature may be construed in a narrower sense as referring primarily to the region in which the laws of quantity and number find marked prominence and elaboration. While the relations of magnitude are not themselves to be mistaken for nature, the sphere of the universe especially characterized by such operations may be taken as such.

At bottom, the meaning of nature seems traceable to the most elementary logical distinctions; apparently it goes back to the basic notions of identity and difference, the One and the Many. Wherever thought stirs and turns back upon itself, the "this" and the "not-this" are distinguished; furthermore, with this apprehension plurality appears. At the same time, each discriminated element is nothing other than just what it is. That is to say, each enjoys a formal identity with itself in the same sense as every other; hence all share in a certain common character (or abstract oneness) despite their separateness (or manyness). Moreover, through these dual aspects of sameness and distinctness, entities come to be envisaged as groups or classes; and, wherever entities are conceived as

a collection of determinate constitution, it is theoretically possible to calculate the arrangements into which they can enter with each other. Consequently, where plurality and difference are emphasized, a logic of the Many tends to develop (in contrast to that of Identity), which takes the form of a theory of combinations and permutations and appears as the distinctive logic of the existential order.

It is, in brief, impossible to conceive the world as an absolutely undifferentiated unity: at the least it must harbor the laws of thought and be divisible into parts enjoying the pure form of difference. Yet this means that wherever logic is applicable there is quantity; for the essentials of quantity are discoverable wherever elements are discriminated from one another on the basis of merely formal distinctions of identity and difference, of whole and part, of more and less. But, although reflection cannot deal with its material in any precise fashion without treating it as unities or collections of unities, there is a marked separation in degree between the vague intensive use of quantity expressed in terms of "more" or "less," "some" or "all," and

that of extensive magnitude, which answers the question "How much?" in respect of an exact number of units. That part of the universe to which the latter methods apply with the greatest success and in developed mathematical form, may be referred to here in a special sense as nature.

III: Probability and Logical Atomism

Wherever there is a collection of entities of limited variety, it ought to be ideally possible, as we have said, to estimate by a process of logical deduction all the dispositions into which they could enter with one another. Furthermore, the same considerations should also permit the computation of the number of ways in which any particular grouping of elements (generally called an "event") might come about, relative to the total number of ways in which it might happen or not happen, a ratio which is said to measure the probability of the event. In other words, in the theory of combinations and permutations, it would seem, we ought to have at hand a method of knowledge whereby, on the one side, the complex arrange-

ments of entities could be analyzed into their components, while, on the other, future events might be predicted and anticipated to an extent at least through the skilful use of the calculus of chances.

Putting aside for the moment the enormous difficulties involved in the application of such a method, one of the most interesting things to note in regard to it is its presupposition of logical atomism. By this we mean that acceptance of a mathematical view of nature and analysis as a technique, rests on the assumption that nature is resolvable into quantitative units or abstract entities among which the relevant qualitative differences, if any, are very few. For such atoms it is not necessary to claim substantial simplicity or indivisibility. For, while each enjoys a formal unity with itself in being precisely what it is, this by no means precludes the possibility of its inherent complexity; any more than the identity, for instance, of air as air prevents the possibility of its phenomenal analysis into a mixture of nitrogen and oxygen. For this reason, even though every concrete application of atomism in a material or substantialistic sense

should be proved wrong, atomic research could still be justified as a logical mathematical method—the conception of objects as collections of entities whose internal differences were unimportant compared to their external disposition with respect to one another.

Thus, at the present time, although no substance has as yet been found that permanently resists the power of analysis, there is a widespread impression that by conceiving solids, liquids, gases, electricity, and energy as aggregates of units of charges, the laws of science can be made to apply to nature more simply and intelligibly than in any other way so far suggested. In line with this tendency, there have developed the quantum theory of energy, the atomic theory of electricity, the dynamical theory of gases, in brief, the whole kinetic theory of matter, together with a wide variety of hypotheses in chemistry and biology, where atomistic conceptions may be traced through Mendelianism down to the latest speculations on units of heredity. Indeed, if atomism be taken in its widest sense as signifying no more than the assumption that nature, if it is to yield knowledge,

must be conceived as composed of units of magnitude, it seems not too much to say that atomism is everywhere a postulate of scientific method, in so far as science attempts to derive measurements, equations, probabilities—in short, mathematical results—from the study of the world. Only by adopting something like an atomistic hypothesis does it seem possible to associate existential states with reckonable numbers and to treat them by definite quantitative methods. In fact, there are some who go so far as to predict, observing the acceleration of knowledge in the last centuries, that the thorough-going application of atomic research to various fields will in the end reveal hidden unities in nature to the extent that knowledge will become systematized into a single quantitative study of combinations in which the particular sciences appear as special phases.

On the other hand, such optimism as the foregoing regarding the future comprehension of nature seems preposterous when one stops to consider the extraordinary, unimaginable variety of ways in which entities can combine; not to mention the weakness of our faculties and

instruments in deciphering even simple combinations. In order to arrive at anything like accurate calculations and predictions regarding a complex state of affairs, it is desirable to know in advance the number of units involved; while, when we are working back inversely from the event to its constituent elements, we ought to know the range of equally likely and mutually exclusive possibilities that condition the event. Yet, as a matter of fact, our situation in confronting nature is precisely the reverse of this. We find ourselves plunged *in medias res,* amid apparently countless numbers and varieties of entities so intricately commingled and obscurely related as all but to baffle our fondest hopes of ever being able to discriminate them. Faced with this appalling tangle of a subject-matter largely inaccessible to control by experimental methods, we seem almost driven, whether we will or no, to take refuge in some deductive theory. And here the analogy closest at hand, and the one which appears most acceptable to reason, is that of the game of chance. We face the world, as has well been said, as if it were an unending lottery from which we were obliged to draw hand-

fuls of tickets in succession. By carefully noting the character, combinations, and repetitions of our drawings, we somehow hope to be able to deduce the contents of the box as a whole. Yet, when we pause to reflect upon the inexhaustibility of the phenomena and the bewildering diversity of orders possible to them, the chances of our ever being able to penetrate the secret of nature's constitution seem pitiably small.

In the face of all this, however, despite the narrow span of our sense-organs and the weakness of our intelligence, this one clue remains as our hope of threading the labyrinth. That is, we still somehow have confidence in the efficacy of treating nature as the permutations and combinations of a small number of elements. The assumptions of the game of hazard seem our best guide; for by treating nature as a combination of relatively qualityless entities in calculable relations, and by attempting to correlate the empirical records of things with theoretical predictions as to their probability, we are at least still relying on the logical structure of phenomena.

In other words, our assurance as to the reliability of this technique is rooted in our knowledge that it is based upon the general laws of logic and their quantitative elaboration. For at bottom, "the theory of probabilities is simply," as C. S. Peirce has said, "the science of logic quantitatively treated."[3] Even when we hazard predictions on the basis of statistical data and proportional frequencies regarding experience, our mathematical expectations are ultimately derived by an inverse process from an *a priori* calculus. In forecasting the future, we are always guided by the assumption that the situation is reducible to a limited number of equally likely, independent alternatives; and our belief in the method is strengthened not only by the fact that it appears impossible to conceive a world that should not conform to the laws of permutations and combinations, but, in addition, by the many results in the physical sciences which appear to exemplify the quantitative constitution of nature. The quotation from Jevons which follows, substantially reflects and summarizes this attitude:

[3] Peirce, C. S., *Chance, Love, and Logic*, p. 64.

In the material framework of this world, substances and forces present themselves in definite and stable combinations. Things are not in perpetual flux, as ancient philosophers held. Element remains element; iron changes not into gold. With suitable precautions we can calculate upon finding the same thing again endowed with the same properties. The constituents of the globe, indeed, appear in almost endless combinations; but each combination bears its fixed character, and when resolved is found to be the compound of definite substances. . . . The wider our experience, . . . the greater the accumulation of well-reasoned knowledge, the fewer in all probability will be the failures of inference compared with the successes. . . . Other worlds and stellar systems may be almost incomprehensibly different from ours in magnitude, condition, and disposition of parts, and yet we detect there the same elements of which our own limbs are composed. The same natural laws can be detected in operation in every part of the universe within the scope of our instruments; and doubtless these laws are obeyed irrespective of distance, time, and circumstance.[4]

[4] Jevons, W. S., *The Principles of Sciences,* 3rd ed., pp. 2-3. Despite the amazing advances in science since Jevons wrote (e.g., the discovery of the transmutation of the elements through the phenomena of radioactivity and the new facts of quantum physics), the burden of his statement remains substantially sound.

IV: Summary

The realm of nature has been distinguished above from that of purely formal relations: first, as being that to which these relations are applicable; and, second, if nature be taken in a more limited sense, as the region especially characterized by "givenness" and to which quantitative and mathematical relations seem peculiarly adapted. Whereas both worlds are acknowledged as logical, the former is characterized by rigorously universal connections, while the latter is marked by the prevalence of disjunctions, alternatives, and notions of plurality and magnitude. In other words, although both general common logic and the deductive theory of probability are alike apodictic, the nature of the second is always essentially disjunctive; whereas that of the first is only partially and sometimes so. And this disjunctive feature in the second unquestionably increases the element of indeterminateness. For, although the theory of probability is itself by no means merely probable, it leaves the reasoner in a state of ignorance and incertitude in any given situation as to which of a number of alternatives will follow.

Furthermore, while both types of logic run the risk of being mistaken as regards the content of their original assumptions, the probability-logic assumes additional liabilities in undertaking to enumerate an exhaustive list of alternatives. All this, however, is mere preliminary to the real business of probability-theory which is that of estimating the value of different combinations of chances. Quantity emerges here in the logic of this theory in the weighing of reasons against each other, in the equating of entities to units, and in balancing events *pro* and *con;* while number also becomes explicit where items are set off in one-to-one correspondence against each other.

It is an interesting fact that the same kind of formal certitude attaches to both these types of reasoning. Whether I infer that a well-balanced die fairly thrown is equally likely to come up a 1, 2, 3, 4, 5, or 6 spot, or that, since mammals breathe air, whales as mammals must do so—in either case, my deduction is characterized by apodictic connections. But if, on the other hand, I decide to reason instead from the particular to the general (i.e., to claim that because

some whales breathe and *some* dice fall ace uppermost about one-sixth of the time, therefore *all* must), in this event there is a marked diminution of certitude observable alike in both processes. In such cases, we seem to infer that, because certain individual facts of experience have, or tend to have, a given character, therefore the class as a whole to which they belong must have this same character. But, in reaching this conclusion, a "leap of faith" appears to be necessary alike in empirical probabilities and in common induction; and, since the results are held to apply beyond observed experience and the data on which they are founded, a degree of uncertainty presumably attaches to the whole transaction. In the following section, we shall consider the nature of induction and the question whether or not it involves such an ungrounded leap.

V: Probability as the Basis of Induction

In an earlier chapter, we expressed ourselves as of the opinion that induction is inverse deduction. In other words, even our thinking about

particulars is always under the control of some tentative universal or latent major premise; and our aim in collecting concrete data is rather to test this universal major by subsuming crucial cases under it, than any notion that the mere multiplication of instances may heighten certainty. In the series of investigations that led to the discovery of the mechanical equivalent of heat, for instance, successive inquirers were dominated by the general idea that "heat is a form of motion" long before it was possible to confirm this hypothesis by exact observation and particular experiments showing that when motion is introduced into an otherwise isolated system a corresponding amount of heat appears (and that when it is not introduced, heat does not appear). But this whole explanation of induction as inverse deduction may still seem to many, despite what was said earlier, to leave unsolved the problem as to what entitles us to assume the tentative universal or major premise in the first place. By what right do we translate our individual experiences into illustrations of a general law? The entire difficulty of passing from *some* to *all* is bound up with this is-

sue of our right to the universal. Hence, this is the question that we must particularly try to answer.

In addition to the uniformity of nature and the postulate of simplicity or system, which have already been mentioned as lying at the basis of natural knowledge, there is also another extremely important notion sometimes referred to as the "principle of induction." Stated in the simplest terms, this principle declares that "the oftener things are found together, the more probable it is that they will go together." At first glance, this may look like nothing more or less than an expression of the empirical law of association. But closer examination reveals its basis in *a priori* theory. Its import is that repeated correlations or sequences in our experience of phenomena may be taken as rough indications of deductive probability, expressing the ratio between the number of ways in which an event can happen and the total number in which it can theoretically happen or fail to happen.

Thus, when we predict the likelihood of a close correlation (or causal sequence) between

friction and heat or lightning and thunder in the future, on the basis of the fact that they have gone together in our past limited experience, we are not relying merely upon blind animal faith or a natural habit of expectation engendered in the organism through the repetition of stimuli, but upon the deductive principle that the members of kinds that are actually more numerous in a large varied collection have a greater number of ways of combining, and hence more chances of occurring than those members of kinds that are less numerous in the whole collection. And, since this mathematical necessity holds not only of the world of experience but of the non-experienced world, we are led to reason inversely that what occurs most often in experience is also most numerous in the whole collection of nature, and that what is less frequent here is also less frequent there. In other words, we recognize that some events are more probable than others because certain classes of combinations have more members than others; and, furthermore, we assume that what occurs most often in experience is likely to be what is most probable in nature, so that we take our most

frequent experiences as a logical (although partial) representation of the whole.

Moreover, as the number of concomitants noted increases, the probability of their occurring in further cases is held to increase; for although no exact correlation can ordinarily be established in detail between the number of instances observed and the degree of probability of the induction, it is generally admitted that the larger the range of "samples" examined, the greater the chances that our predictions will be fulfilled. The truth of this may be illustrated as follows. If the number of observations already made is *n,* and the number of observed correlations between two given phenomena is the same *(n),* the likelihood that the association will occur in the next observation is to be estimated as $(n + 1)/(n + 2)$. (That is, the ratio is calculated as the number of favorable cases of the event's happening, plus one more possibility, divided by the total number of happenings with the favorable and unfavorable chances of one more case added to it.) Thus, as the number of concomitants observed increases, the fraction will grow larger, while the probability

of the same association's occurring in the next case approaches nearer and nearer to unity (1/1) or certitude.

The great problem in dealing with nature is, of course, to find a means of compassing her infinite variety and myriad forms. Faced with her inexhaustible fecundity and the endless succession of individuals that embody every type (bacteria, dew-drops, stars, numbers), the range of possible experience and knowledge by direct acquaintance shrinks to a mere pin-point in comparison. Oppressed by an overpowering subject-matter largely beyond the reach of experimental methods, we are, as we have said, almost inevitably forced to have recourse to deductive theory, although our assurance as to its adequacy to the data is purely formal. Yet, although the difficulties of the method are often too great for our powers, at least we cannot doubt its universality of scope or that the perplexities that arise are due rather to our own lack of skill than to anything inherent in the relations of the subject-matter.

Thus, if these brief suggestions are not mistaken, the basis of the transition from some to

all, and of our right to appropriate universal premises in dealing with nature, is not an ungrounded leap of faith but the application to phenomena of the formal principles of combinations and probability. And once considerations of probability are recognized as fundamental, the process of "sampling" or what might be called logical representation of whole by part becomes central in the method of induction. When, for instance, we estimate the prospects of an unopened mine by picking up specimens of rock or ore scattered in the vicinity, or when we sample a carload of wheat by examining handfuls from different parts of the shipment, we rely upon the principle that smaller groups, if chosen impartially and at random from a larger group, tend to have the character of the larger group. Having discovered from the probability-curve that the combinations or chances which give us the mean character of a collection are more numerous than those which represent its extremes, we naturally conclude that those examples which appear most frequently in experience are more likely to depict the dominant character of the whole than any

others. This same idea of representation of whole by part, as we have said, governs our predictions regarding causal sequences. That is, the associations of phenomena that occur with the greatest frequency in the "sampling" (or the smaller group of events that marks our experience) are taken as tending, as a rule, to provide a correct description of the correlations that characterize the larger, unexperienced group of natural phenomena. And, although the method of sampling is never beyond risk of error, its mistakes often cancel out; and the process tends to better itself. Broadly speaking, the larger the range and number of samples, the smaller the margin of error; the greater the proportion of a given type of correlation, the greater the chances that we have discovered a genuine law of nature. But once more, in conclusion, it should be pointed out that the method of sampling does not rely fundamentally upon experience as experience, or upon the simple induction *per enumerationem simplicem* of Mill, but rather rests upon the laws of probability and the interpretation of concretely observed cases as illustrations of them.

Perhaps the analysis of a simple example of sampling may serve to bring home more clearly its assumption that the members of a kind (or sub-group) that is more numerous have a greater number of ways of combining (and hence better chances of being chosen) than have the members of a kind that is less numerous in a varied collection. Suppose, for instance, that I have five apples in my pocket, of which two (*p* and *q*) are red, and three (*x*, *y*, and *z*) are green. In such a case, if I undertook to draw them out two at a time (replacing them after each drawing) and attempted to judge the color and proportions of the apples in the collection on the basis of these drawings, what would be the chances of my judging right rather than wrong? In the first place, the number of possible combinations by pairs would be ten; that is, there could be three pairs of green apples (*xy, xz, yz*), one pair of red apples (*pq*), and six pairs composed of one of either color (*px, qx, py, qy, pz, qz*). Accordingly, if all possible pairs were drawn successively and independently, a preponderance of the judgments would correctly describe the collection as a

mixed one, since the mixed couples would outnumber the unmixed couples in the ratio 3/2. Furthermore, the chances of my drawing a mixed couple (i.e., a sample of the largest subgroup, which ought therefore to have the most ways of combining) would be six-tenths or three-fifths; while my chances of drawing a pair of green apples at any given draw would be only three-tenths, and that of a couple of red apples (or samples of the smallest sub-group), one-tenth.

Broadly transposing the figure, we may conceive of nature herself as the bulging pocket and man as the judicious sampler who is forced to draw forth combinations of phenomena from the hidden depths of the *rerum natura;* for, at his peril and in order to survive, man is compelled to draw general conclusions from his meager experience, and to predict the future in the light of his restricted past. And everywhere, whether he predicts crops according to the vague lore of common sense or the birthrate on the basis of social statistics, or derives the gravitational correlation of space, time, and mass from the results of a few experiments, he

relies upon the same principle of the valid substitution of the part (or sample) for the whole, and the notion that what happens most frequently in experience is his most probable or best clue to the nature of nature in that respect.

But an objection is sure to be offered at this point by certain students of probability. Granted the justice of the assumption that what there is more of in nature has more ways of combining, and therefore *ideally* more ways of happening than what there is less of, this does not in the least prove, they will say, that actually *in experience* what there is most of really will happen most often. In other words, on this view, a *saltus mortalis* exists between the theoretical probabilities in question and the actual statistical frequency-ratios gleaned from experience. In other words of Venn:

> How do we know that cases which have the most numerous causes, that is, which can be brought about in the greatest variety of different ways, will in consequence occur with most frequency, except upon grounds of experience?[5]

Although it is usually admitted that the prac-

[5] Venn, J., *The Logic of Chance*, 2nd ed., p. 94.

tical science of empirical probabilities and statistics depends upon there being a close correlation between the purely logico-mathematical theory and the actual behavior of things, no satisfactory explanation of this correspondence can really, on this view, be given. Why should the *a priori* and *a posteriori* keep step with each other? The only justification that can be offered, according to the empiricists, for our continued confidence that the results of experience and theory will generally coincide, is that derived from the record of experience itself. It is only, they say, because past experience has shown us that empirical frequency-ratios tend in the long run to approximate to the demands of deductive mathematical theory, that we are entitled to believe that they will correspond in the future. But this, as it seems to us, is just the old empirical fallacy over again of arguing that "the future must resemble the past." To maintain that "What has been must be, because it has been," is equivalent to saying, "*A* must be *B*, for the reason that *A* is *A*." Plainly, a begging of the question is involved where the effort to bridge the gap between the experienced (past)

and the non-experienced (future) is argued solely on the testimony of experience itself.

As we see it, the burden of proof rests rather upon the objectors. That the statistical results derived from such data as ore, wheat, stars, deaths, and the molecular movements of gases should, on the whole, tend to conform to the *a priori* requirements of combinations and probabilities is presupposed, as it seems to us, in our ultimate assumption that the universe is logical. The mere fact that the results of *a priori* theory cannot always be shown to be exactly confirmed in experience in the particular case, need not alter the necessity of the laws or their supremacy over empirical data, any more than the fact that the seemingly improbable happens at every moment, can be said to constitute a violation of the laws of probability.[6]

Yet, while the universal applicability of the general principles remains unchallengeable, a gulf appears between what is theoretically conceivable by means of them and what is prac-

[6] From the point of view of the universe-at-large, with its endless variety of possible systems and arrangements, the probability of the occurrence of one event rather than another would appear to be $1/\infty$

tically attainable through our limited resources of direct acquaintance. We may know that a given statement is correct or incorrect (e.g., that there either are or are not canals on Mars), and yet not know which alternative is really the more probable. We may know that the chances of a penny's falling heads or tails at every throw are theoretically 50/50; yet in the actual case be uncertain whether the possibilities are equally likely or that one throw is independent of another. We cannot be sure that the given penny is a perfect coin with equal, circular sides and its center of gravity in the middle; or that it is tossed strictly "at random" each time with either side held alternately upward in the palm and without any favoring twist of the wrist. Because of this ineradicable ignorance of conditions governing the particular case, our difficulties regarding the relation of fact and theory can never apparently be wholly removed. Nevertheless, the fact that, in an extensive series of trials, the results of practice usually approximate quite closely to the requirements of theory, may be taken to illustrate the possibility that they can be reconciled in principle.

In dealing with more complicated situations than that of the simple game of chance, the effort to formulate the demands of mathematical theory and to test facts by it becomes immensely more difficult. In most cases, it is virtually impossible to estimate all the possibilities involved or to know in advance that they are independent of one another. In calculating, for instance, the death-rate from respiratory diseases in a given year and state, it is practically certain that many cases of mal-diagnosis and the complication of diseases figure in the statistics, that not all deaths have been recorded, and so on. It ought not to be surprising, therefore, considering the limitations of our knowledge of particulars, that empirical results and mathematical prediction rarely coincide exactly in the individual case. At the same time, since we know of the existence of these restrictions and difficulties, the discrepancy can hardly be construed as refuting any possible conformity under more favorable conditions. A contributing factor to the divergence may also be the opposite ways in which the results of fact and theory are obtained: statistical frequencies take their start

from the *fait accompli*—from a plurality of complex events that have already happened—and attempt to estimate from the proportion of given coincidences in these events the likelihood of similar occurrences in future; whereas *a priori* theory, on the other hand, is directed upon a single ideal event in relation to a system and the ratio of the number of ways it could conceivably come about compared to the logical totality of ways in which it might happen or fail to happen.

VI: Probability Not Derived from Mere Ignorance of Impossibility

Nor is the theory of probability merely a schema devised by us to trade upon our ignorance. A fundamental error is made, in our opinion, by those who locate probability either in the *minds* that hold certain beliefs or in the *things* or *events* that are brought about. Rather, its principles are bound up with the logical structure of the universe and, as such, hold irrespective of actual knowledge or ignorance. When conceived as a mere intermediary state of knowing midway between truth and falsity,

probability was earlier seen to involve a destructive paradox. For there is a contradiction in assuming that a given proposition may be *either true or false* (since we do not know which it is) if, at the same time, we assume, as we must, that the proposition is *true and not false* (since every proposition assumes its own truth). The conclusion to be drawn, apparently, is that probability as such cannot be located either in our knowledge of the proposition or in the proposition itself, but is rather in the nature of a proportion that subsists between a certain body of evidence and the total number of possibilities inherent in a particular field.

Only too frequently, however, probability is construed subjectively as belonging wholly to particular minds. De Morgan,[7] for instance, makes it identical with "degree of belief"; while Jevons[8] likewise conceives it as wholly mental, although stipulating that it measures belief not as it actually is but as it ought to be in a given mind on the basis of the evidence present to it. Such views diverge to a greater or less extent

[7] De Morgan, A., *Formal Logic,* p. 172.

[8] Jevons, W. S., *Principles of Science,* 3rd ed., p. 198.

from the logico-mathematical view of probability which interprets it in terms neither of thoughts nor of actual external states of affairs, but as the ideally deducible ratio between the number of ways in which a certain configuration of elements could occur, relative to the total number of conceivable dispositions in accordance with which it could occur or not occur.

Take the familiar instance of an aeroplane or dirigible that sets out on a cruise of the air and that fails to reach its destination within an allotted time. In such a case, the outcome of the voyage will undoubtedly be estimated with varying degrees of probability by different minds, according to their tendencies to belief and the evidence obtainable by them. Thus, although there is no real uncertainty in the event itself, since the airship and crew have either been destroyed or have not been, there may be much in the minds speculating upon it; and it is this fact that has led many to refer probability wholly to the mind. But to us, this view must be rejected, for the reason that where probability is made dependent upon particular minds, a factor of subjective interpretation, mood,

suggestibility, native capacity or the lack of it enters, all of which tend to destroy any possible common or objective basis of computation.

A typical instance of the kind of error arising from the subjective view of probability is to be found in the assertion that complete absence of knowledge regarding any proposition (e.g., that "There are black beetles on the planet Venus") constitutes a 50/50 probability of its correctness or incorrectness. Thus, since by the law of excluded middle, the airship has either been destroyed or not been destroyed, and since the watchers are ignorant of what has happened, it is estimated that there is an equality of chances each way. This is the interpretation of probability as ignorance of impossibility, which calculates on the basis of there being no information either *pro* or *con,* that the chances are 50/50. Yet, to suppose that alternatives concerning which we are ignorant are, for that very reason, equally likely, is to assume, as Keynes points out,[9] that any proposition asserted at random or any answer to any question is as likely as not to be correct. But obviously

[9] Keynes, J. M., *Treatise on Probability,* p. 182.

this is not so. And, although our mistaken application of probability is here subjective, this does not make the probability involved itself subjective, any more than a mistake in arithmetic proves an error in the arithmetical system.

A more satisfactory view of probability would recognize, if we are not mistaken, that it is inherent neither in the subject nor in the object of knowledge, but is to be found instead in the logical relationship of terms to each other. In the case of the airship, for instance, there is certainly a body of data conceivable which could furnish the basis for ideal computations of probability as to its fate, even though no actual inquiries were ever instituted by particular minds. Such facts as the presence of storms in that general region at the time, the velocity of the wind, the temperature and atmospheric pressure, the condition of the engine, and the supply of fuel carried, would constitute a body of data which could theoretically be combined in different ways to produce different results; and even though every one remained ignorant of the actual number of constituent factors in

the situation and their possible combinations with each other, no one could doubt that they must have had some definite objective constitution despite our lack of particular knowledge in regard to them. Furthermore, it must be assumed that, from any conceivable standpoint of perspective, there would be certain clues outstanding, which, when compared with the totality of positive and negative clues (or combinations and permutations of elements) available from all possible points of view, might be said to constitute the probability-ratio that obtained relative to these given conditions. The probability of an event is thus, in our opinion, always with reference to a specific point of view or, in other words, is a relation between two ideal groups of evidence, the one available from a certain standpoint and the other available from all possible standpoints.

CHAPTER VIII

THE PROBLEM OF TRUTH

I: Truth is Not Exact Correspondence

OUR discussion of nature and inference in the previous chapters has brought another problem to the foreground. This is the problem of truth. So far, it has by no means been made clear whether truth consists wholly in formally valid reasoning or whether there is, in addition, a truth that appertains to the content, of a material character. To the question, no answer has been given, although, on the other hand, indications have not been lacking that truth cannot be regarded as in any sense a mere property of objects or as belonging to purely sensuous facts. In attempting here to understand what truth is, it may be of value to begin by acquiring some comprehension of what it is not.

The every-day judgment of common sense, together with that of a large part of scientific theory, seems to be to the effect that truth rests in some kind of correspondence; and there is considerable historical likelihood, apparently,

that under this more sophisticated guise the primitive copy-theory survives to our own day. Few, it goes without saying, would be so hardy as to maintain nowadays that the "true" account of an event is its representation by a copy that reproduces it as literally as the mirror the face; yet the motive which construes truth in this way lives on and gains an outlet through many channels. But, even in the case of mirroring, the critical mind is forced to recognize elements that prevent the copy from being an *idem alter* of the original; not only is the order of the parts in the reflected image the reverse of that of the original, but the reproduction also occurs in a different medium. Obviously, with the relinquishment of an identical material, the possibility of literal and exhaustive reduplication is irrevocably lost. There remain, however, varying degrees of literalism; and the process of abstraction and renunciation is carried to different lengths in different fields of experience. A motion, as described in the symbols of mathematics, for example, cannot rival in closeness of "likeness" the pictorial representation of the same motion; nor can the ideal formula for a body

claim the same matter-of-fact exactness in reproduction as a physical model constructed to size and scale. Yet all these distinctions in degree in no wise affect the fundamental point that the moment the necessity of full, literal reduplication in the same medium is remitted from our notion of truth, the copy-theory is abandoned in principle. The attempt faithfully to picture the original gives place to an effort to create a representative, the primary function of which shall be to "stand for" the original and to serve the purposes of a surrogate. But when this happens, when the aim of complete *reduplication* is supplanted by that of *substitution,* and the childlike desire for sensuous reiteration is replaced by the motive of practical or theoretical usefulness, then and only then does the copy-theory give way to the notion of correspondence.

To expound the correspondence-theory and other theories, with a view to according them a show of fair treatment before passing on to consider their inadequacies, is not our purpose. On the contrary, since our main concern is to develop a special view of truth, rival theories must be treated to short shrift, with the proviso

only that the deficiencies that lead thought to prefer another position to them, shall be indicated. The first difficulty that strikes us forcibly in regard to the correspondence-theory is that what is meant by "correspondence" is not made clear. Shall we say that correspondence is to be found in the mere tallying-off of the parts of two entities against each other point by point, so that the truth of one with respect to the other consists in this mere one-for-oneness? Or is a certain likeness of a sensuous sort also required? If I say that a graphophone record bears a true representation of a certain musical composition registered in the grooves of the disk, or if I say that a map accurately depicts a country, in both cases the reference is apparently to the first kind of correspondence; whereas, if I refer to the portrait of a friend as "true to life," I seem to indicate a second and different sort of correspondence. That these two kinds are not synonymous and interchangeable is as obvious as that a portrait done on the principles of map-making (point-for-point correspondence of relative positions) could have no truth as portraiture.

Another troublesome point appears in the question as to what degree of correspondence is necessary to constitute truth. In the consonance between portrait and sitter, or between language and the objects described by it, exhaustive correlation is out of the question; yet this apparently offers no bar to their possible truthfulness. But, if correspondence does not need to be complete, *at what point* can truth be held to supervene? Just how much or how little concurrence will suffice for the attainment of verity? Lastly, because correspondence seems to stand forever on the threshold bidding adieu to the notion of literal picturing (as where it is reduced to the similar ordering of points in space) yet is never able to rise wholly to the level of ideal substitution— for this reason there seems something baffling and equivocal about the position.

But the most serious trouble with the theory is epistemological, and arises from the fact that concurrence between an idea or proposition and reality (in which correspondence is generally said to consist) cannot be tested by the criterion of correspondence. How can we compare ideas

with reality? Apparently, such comparison could subsist only if things-in-themselves and ideas were both immediately present to us and able to be viewed independently of one another; but this is impossible, since we have no power of standing outside the circle of our ideas or of scrutinizing reality *per se* and notions separately.

A slightly varied form of the correspondence-theory describes truth as constituted by the concurrence of an object with its meaning. Under these conditions, a proposition respecting some physical state of affairs is said to be true when the meaning that it affirms with reference to the object coincides with the actual object as perceivable. Very different meanings may, however, presumably be imputed to the same object. The same moonlight, for instance, may be taken to be yellow by the farmer and green by the artist; and to ascertain which one is right appears next to impossible. Or you may assert that your neighbor's cat is a gray cat, and claim to have tested the truth of your proposition by observing the cat itself. Nevertheless, if you consult other observers, they may tell

you that the cat is yellow and that you are color-blind. In such a case, even though there might seem to be genuine agreement between the *object as meant by you* and the *object which you observed* (so that *for you* a gray cat was the "true" cat), still a contradiction would remain between your individual verdict and that of the majority of observers.

The question would then arise as to which authority you should accept as true, the individual or the social; and neither alternative appears to be entirely acceptable. For if, on the one hand, truth is held to be a function of the concurrent meanings and perceptions of the majority, then, plainly, quantity and the weight of numbers enter to supplement correspondence as the test of truth; whereas if, on the other hand, the agreement of the private, individual experience with itself is taken as the mark, in that case we may expect as many varieties of truth as there are individuals with peculiar organisms. In the second place, whereas the truth claimed *in the proposition* "The cat is a gray cat for me" appears purely individual, that alleged *for the proposition* is of a superindividual kind. In

other words, a contradiction is implied in our very utterance, since it claims a superindividual sanction for individualism in asking others to concede that my private experience concurs with itself; whereas obviously others (not having a trap-door to my mind by which to compare my percepts and meanings) have no reason to admit the truth of my assertion even as a description of my own experience. Apparently, therefore, the notion of truth as agreement between our meanings and perceptions would have to be rejected, both in its individual and social form; for, although meanings are generally common and communicable, no substantial results can be obtained from a view that requires their correspondence with perceptual objects, which are in their nature private and partly physiological.

Owing to problems like these, correspondence must be rejected as a theory of truth. Only by assuming the existence of a type of true knowledge distinct from correspondence could we know correspondence when we had it. For our assurance as to the nature of truth has to be purchased by means of a criterion already, by

some other means, known to be true. Hence, unless we presuppose that we already possess some infallible test for distinguishing correspondence from the lack of it, and for identifying the former with truth, we cannot claim to know in what truth consists. Thus, an epistemological devil is found lurking in what at first looked like an extra-mental view of truth.

After all, we can only compare ideas with ideas. And the reduplication of experiences by propositions constitutes at best only a vague and partial mark of verity; for, although it provides an external "sign" that is sometimes useful, it sheds little light on the inner constitution of truth. None the less, the recognition that representation must be symbolical, and the substitution of signs for pictorial copying, has its value as marking a growing apprehension of the importance of universals as opposed to the particulars of sense.

II: Truth Not Identical with Self-Evidence

But once the correspondence theory is discarded, the number of likely possibilities is

greatly reduced. Among those remaining, the views regarding self-evidence deserve special mention. Of these, there are two main types corresponding to the two classes of objects for which immediate certitude is commonly claimed. According to the first, truth is rational self-evidence or what is known by intellectual intuition. Certain ideal objects of the intelligence (axioms, principles, and certain laws) are said to shine in their own light with an immediate luminosity that cannot be gainsaid. At the same time, their power over the mind is held not to be derived from any feeling, emotion, or instinct, but rather from a capacity for intuitive intellectual insight by which the mind discerns certain things without discursive effort. A group of principles supplied with this power to awaken immediate conviction stands, according to this account, at the basis of knowledge. Yet these axioms do not comprise the whole of truth. In addition, there are also secondary truths of demonstration which, while not themselves intuitively certain, are yet capable of being construed as logical consequents of "first principles." In the words of Descartes, "many

things are known with certainty though not by themselves evident, but only deduced from true and known principles by the continuous and uninterrupted action of a mind that has a clear vision of each step in the process."[1] Though not themselves self-evident, these secondary truths are conceived as endowed with a derivative certainty that is absolute.

Admittedly, there is much to be said for this type of self-evidence. Truth does seem to shine in its own light without requiring illumination from any other quarter. And so much is this the case sometimes, that elaborate proof seems not only powerless to strengthen conviction but in the nature of a profanation. The assertion that $2 + 2 = 4$, or that every effect has a cause, or that two sides of a triangle are greater than the third, appear possessed of this higher immediate obviousness. Still, we must be on our guard against the mistake of acknowledging logical necessity where it is not present. Much more is subjectively certain in this world than is logically necessary. And it is therefore advisable to search for some mark that will clearly dis-

[1] Descartes, *Regulae*, III.

tinguish self-evident objects of reflection from those that merely gratify our strong natural inclination to belief. A start is made in this direction, of course, when it is noted that self-evident propositions enjoy almost universal acceptance and that they seemingly impress one and the same meaning upon different intelligences. Yet, although this mark serves fairly well to differentiate what is "rationally certain" from the "irrationally given," it fails in the narrower sense to distinguish the self-evident, which is immediate, from the mediate forms of the rational known as truths of demonstration. To do this, it seems necessary for us to fall back upon the word "intuitive." What distinguishes self-evident propositions from the rest of knowledge is, after all, their intuitive character; accordingly, the crucial issue of self-evidence hangs on the meaning of intuition.

The chief question that needs to be asked in regard to these intuitive truths is whether they can be reduced by any means to terms of analytic or discursive reflection. Of the two answers possible, the one less likely to be made is "yes": less likely because in that event it would be hard

to show how they differed from ordinary demonstrative truths grown psychologically immediate through familiarity. For, if rational intuitions harbor nothing in their essence really evasive of capture by mediate means—nothing finally superanalytic or subanalytic—what ground remains for urging that they are indemonstrable? On the other hand, if the exponent of self-evidence answers the question in the negative and maintains, as he is likely to do, that the essence of our *intellectuellen Anschauungen* cannot be presented in discursive terms, he chooses the stronger position, the one harder to meet with argument.

Still, as it seems to us, there are several objections that weigh heavily against intuitionism even in this form. The first one is that many seemingly unanalyzable intuitive insights or axioms turn out to be merely postulates of familiar systems, and lose their "self-evidence" the moment that we pass to other systems. Thus, the postulate that "a straight line is the shortest distance between two points" need not hold for non-Euclidian systems. Again, many "truths" that appear intuitive and underived at

first glance are later seen to involve a complex intellectual operation in their apprehension. For instance, although the truth that $2 + 2 = 4$ may seem instantaneously obvious, the insight that it is legitimate to pass from one side of the equation to the other implies not only recognition that $2 + 2$ is a sufficient ground of 4, but also the notions of number, addition, and equality. Owing, however, to the great familiarity and simplicity of the operation of addition with such small numbers, its representation in consciousness has become psychologically abridged; and the necessity that properly attaches to the logical process as a whole comes to be associated with the mere apprehension of the words. The last and greatest weakness of intuitionism, in our opinion, is the claim that knowledge at bottom rests upon certain primitive, underived truths immediately apprehended. For to suppose that knowledge is ultimately based upon "reasons" for which no reason can be given, is to violate the law of ground and consequent, which requires the assumption of an unbroken chain of logical antecedents and successors. So long as the in-

tuitionist remains a rationalist, he cannot claim finality for the content of his intuitions, since to do so would be to defy the canons of the abstract understanding.

The other main view of self-evidence, to which we now turn, is that self-evidence is irrational and that it appertains to certain facts of direct experience known apart from intellectual insight, such as sensations and feelings. Instead of claiming to convince our reason, these "truths" overcome us by main force. Their self-evidence consists in their being immediately irresistible, and in this frankly irrational sense is so defined.

In the main, the pretensions of such a view have been already examined. How can the mind's inability to resist the incursions of data be equivalent to proof of the authenticity of these data? If so, evidence of truth must be taken to mean brute givenness, which is something very different from what is usually understood by it. That we cannot doubt sensations and feelings at the moment of their occurrence is only because to doubt is actively to repel suggestions before the mind, whereas to

sense or to feel is in so far to accept stimuli passively and responsively. Assuredly, the dead weight of a sensation or feeling is not the same as self-evidence, since they lack the clarity and distinctness that are held to be the special marks of truth. For the latter, the mere psychological feeling of certainty is no adequate substitute; since, as is well known, such feelings form the main stock in trade of abnormal persons haunted by the most extreme aberrations and manias. Mere intensity of conviction, when unsupported by grounds, is no assurance; while that which overwhelms the mind by sheer force is, in our opinion, peculiarly to be mistrusted. For, though such force may storm the senses and batter the will with ceaseless iteration, it can never by such means win from reason the verdict of truth.

Although the exponents of sensuous self-evidence continue to hold the contrary, to us it seems plain that universal, necessary knowledge is not to be found in the content of immediate experience. Nevertheless, at the risk of a certain amount of repetition, we must pause to consider their objections. "Is it not

luminously apparent," they will protest, "that we perceive what we perceive, and that we perceive red as red and not as green? Are not these sensuous truths precisely as irrefragable as the logical truth that *A* is *A* and not *non-A?*" To which we must reply that there is an important difference between them. Whereas, in the case of logical truths, certitude as to their *what* and *that* go together, in perceptual experience the meaning and existence of the data may be dirempted from each other, and neither is really beyond the risk of error.

Elsewhere[2] we have cited instances to illustrate the lack of certitude and discrepancies in our perceptions. How little self-evidence they have is shown wherever we are forced to revise our beliefs as to the content of our perceptions through becoming aware that our meanings and sensations are at variance with one another. For example, if we are in the woods when the sun is momentarily obscured by a cloud, it is quite likely that we shall continue to "perceive" the trees as of the same shade of green although actually we are receiving different

[2] Cf. pp. 165-168.

sensations. If, however, our attention is called to the incongruity here between meaning and sensation, we shall immediately admit our mistake and set about mending the ways of our perceptions. Yet the very admission of a discrepancy is enough to destroy the pretension to infallibility in the experience.

While the assertion that *we perceive what we perceive* is formally true enough, it is analytic and tautological. The error of the intuitionist, as it seems to us, is to attempt to make it over into a synthetic truth by quibbling on the dual aspects of perception: having noted that the proposition *we perceive what we preceive* covers equally well and legitimately both the judgments *we mean what we mean* and *we sense what we sense,* he attempts by a fallacious shift of meaning to assert that *we sense what we mean* and vice versa. But where this is done, the formal certitude that attaches to propositions of the type *a* is *a* is unwarrantably claimed for one of the type *a* is *b;* and the uncritical observer is persuaded to accept as indubitable a synthetic statement of fact, under the impression of its being a purely formal statement.

One of the salutary functions of art is also, as it seems to us, to destroy the illusion of self-evidence. In other words, art shows us how rarely we are experiencing what we think we are, and relieves us of some of our notions as to the immediate obviousness of nature. Painting, especially, serves to awaken the plain man to the fallaciousness of his belief that what he apperceives is unquestionably what he senses. For frequently the meanings that he attaches to his sense-data indicate not only the crudest sort of observation, but are not even consonant with the data themselves. Thus, he will take the moonlight as yellow when he is sensing it as greenish blue. Snow for him will be dead white; and he will marvel helplessly before a picture representing it as splotched with purple, blue, and red. The whole world of chiaroscuro, of atmospheric effects, and illusions of distance (all of which so completely modify the appearances of things) is largely lost upon him. In the midst of what, for the artist, are the fabulous riches of sensory life, the plain man is content to live upon a niggardly pittance of hard, cold, inaccurate meanings.

Yet, if once the observer is awakened to the fact that meanings and the sensations that elicit them are not always accordant with each other, his belief in the self-evidence of perception will be greatly weakened. No longer will Mill's contention go unchallenged that "whatever is known to us by consciousness is known beyond possibility of question" so that "no science is required for the purpose of establishing such truths";[3] or Mr. Russell's statement be admitted that the "truths of perception have the very highest degree of self-evidence."[4] When

[3] Mill, J. S., *A System of Logic, Introd.*, sect. 4.

[4] Russell, B., *Problems of Philosophy*, p. 183. Admission of the self-evidence of perceptions often carries with it acknowledgement of *degrees of self-evidence*. Thus, greater certitude seems to attach to my perception of the present moment than to those that I recall of yesterday. Again, it is said, there may be gradations of self-evidence in successive perceptions. For instance, in listening to a horse trot away along a hard road, the certitude of my perceptions (though each is certain in itself) may be held to diminish successively in degree. But self-evidence so interpreted, in our opinion, means nothing more than grade of intensity of psychological belief. Of certitude in the logical sense there can surely be no degrees—no middle ground between certainty and uncertainty. What degree of self-evidence, we are tempted to ask, attaches to the data at the point at which we do not know whether we are still hearing anything or not? At this stage, at any rate, it should become clear that

this state of affairs comes about, it is time to look for another view of truth.

III: Truth Not Identical with Mere Coherence between Facts and Judgments

The wide-spread favor shown the coherence-theory in recent years, a favor that it presumably enjoys in part through the influence of Kant and the ascendancy of scientific methods, calls for special comment. According to this theory, facts and laws are held mutually to establish each other. On the one hand, principles are proved by their power to explain phenomena; and the phenomena, on their side, are accepted because they bear out the principles. Or, to put the matter in another way, an hypothesis is said to be true because it accounts for the facts; while the facts are declared true because they corroborate the hypothesis. If the data and the hypothesis cohere in scientific investigation so that "everything works as if they were so," they are accepted as *bona fide;* not,

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the evidence was not really inherent in the sense-data, but rather in the belief put upon them.

of course, as beyond the possibility of revision, but as, at least for the time being, the most adequate way of taking the subject-matter. Accordingly, proof, from this point of view, since it consists in the reciprocity and correlativity of facts with theories, agrees with the hypothetical method of the natural sciences.

But the hypothetical method, as has already been pointed out, does not pretend to establish any final truth. The solution to a given problem claims advantage over rival answers only tentatively and provisionally, usually because it embraces a wider range of experience and explains matters more simply (i.e., with fewer assumptions) than they. That is, the amount of truth that it possesses is said to be measured by the inclusiveness and harmoniousness of its range. Yet, at any moment, it is possible that a system of explanation still more harmonious and more inclusive may appear upon the scene —a case in which what was hitherto relatively true may become relatively false. For, in the words of the coherence-theory, "the only test of truth is more truth"; and only the whole of experience is wholly or absolutely true.

In this connection, we may begin with a criticism of the doctrine that the criterion of truth is the whole, and that everything which falls short of the totality of experience, by reason of its merely partial character, can be only partly true. The weakness of the position, if we are not mistaken, lies in the fact that at bottom it is a theory of reality, not of truth. For the standard of truth, in being identified with the sum-total of things, becomes synonymous with the overmastering power of the environment; so that whatever is favored by natural selection, and is able to sustain the combined pressure of the surroundings, is held to be in so far true. Thus, the survival of a thing in the struggle for existence, its hardihood in weathering the sculpturing-process of the whole, is taken not merely as indicative of a happy adaptability and material resilience, but as a *factum* constituting its truth. Although this interpretation is, of course, contingent upon forcing the theory to its logical extremity, nevertheless where truth is identified with the concrete experiential whole as here, nothing else than its assimilation to reality can well be meant.

A rough illustration of the application of the criterion of the whole to a belief, may serve to show its weakness. Suppose that members of an African tribe confront a white man for the first time. How ultimately can they test their belief that this strange being has objective reality? Obviously, if the only test of truth is more truth, and if only the whole of experience is true, they cannot expect to get very far merely by touching him and examining his garments. According to the theory, they should accept whichever belief about him appears to conform to the widest range of experience. And which this is, there is no way of knowing in advance. Accordingly, the process of belief must be allowed to work itself out; and whichever interpretation of the white man (as divine, as an edible, as an apparition, or as an equal) appeals in the long run to the majority of the tribesmen, may be said to be proved true by its survival. In other words, that view which fits in best with the environmental situation and outlasts other views, is proved true by natural selection and the triumph of brute fact.

Clearly, appeal to the whole in this sense of

reliance upon the force of circumstances is vastly different from reliance upon logical system. For if appeal were made to the latter, the problem would take the form of an estimation of probability which is mathematical and *a priori,* instead of remaining a matter of dull submission to the dead weight of fact. That is, the testimony afforded by the sight, touch, and hearing of a number of independent witnesses could be treated as so many elements in calculating the probable truth of the belief in the objective existence of the white man. And if the number of independent observers was large, and if their evidence tended to be mutually corroborative, the logical probability of the truth of their verdict would be considered great.

Other objections to the whole of reality as the criterion of truth may be briefly mentioned. In the first place, if nothing short of the totality of experience could be wholly true, it would follow that perfect knowledge would demand a knowledge of everything. The meaning of a thing could be understood, presumably, only in its complete connections with everything else. Yet when the requirements of truth are made

so difficult as this, truth would seem to become unobtainable. For, since a concrete knowledge of everything is impossible, we plainly cannot claim to know *what truth is* (since we do not possess it) without self-contradiction. The notion, furthermore, that nothing can be really comprehended unless we know its connections with everything, is obviously contrary to experience. We do not have to prove the whole of arithmetic before we can know that $3 \times 2 = 6$, or demonstrate the rest of geometry before we can be assured that the sum of the angles of a triangle is two right angles. Although, in the case of number and measurements, we recognize a certain interdependence of values within the given field, so that each element has its status defined as a function of the others, nevertheless appreciation of the truth of any specific value does not require exhaustive realization in detail of its relation to all the rest.

A second criticism of the coherence-theory has to do with its emphasis upon the organic nature of truth and reality. Too often, in our opinion, talk about the vital interrelatedness of the parts of reality and the harmony of the

whole, cloaks a confusion between the biological and the logical standpoints. Although, in so far as reality is logical, it *ought* to be systematic, and one thing should lead to another, it cannot be shown in detail in actual life how this is so. Again, to say that the norm of logical system or rational nexus governs all valid reflection, is one thing; to say that the universe is like an organism, is quite another. For the latter view is biological and existential, implying an implicit teleology in the nature of things that makes for mutual concordance and harmony. But the existence of any such thorough-going adaptation of means to ends throughout experience cannot be experimentally verified. Accordingly, the organic view with its emphasis upon the concrete fusion of form and matter, must be rejected.

IV: Truth as Equivalent to Valid Logical System

Any attempt to show that content is organically interdependent with form, tends, as we argued earlier, to destroy the universality of truth as holding for all meanings and to attach

it instead to certain particular meanings. For, once content is allowed to occupy a position equal to form and fusible with it, the center of gravity in truth is shifted from control by rational necessity to control by particular existential forces. But, since certitude is never obtainable with respect to the latter, knowledge is thereby reduced to a state of indeterminate probability.

In our own opinion, truth arises not from any concrete blending of fact and theory, but rather where the particular meaning of the content serves as a clear sign or symbol of some formal pattern. For that a thing is true means, we submit, that it holds in a particular case because it holds universally and necessarily; hence its essence is revealed by showing that it is an instance of a general law.

That truth is formal is not to be taken as meaning that it is determined entirely by the natural framework of our minds; nor is certitude identical with what we cannot help thinking because of the constitution of our faculties. Envisaged in this way, truth would be simply equivalent to what we are compelled to believe

by our biological make-up. But, from the beginning, we have sought to combat the fallacy of mistaking for truth what we believe merely because we are impotent to believe otherwise. Natural limitations are not veridical compulsions. Truth, on the contrary, is distinguished from other objects of belief by having in its nature that which convinces the whole rational man, and which does not merely crowd itself upon him by main force. Truth is articulate, not inarticulate, as regards itself; its cogency and discursiveness stand out in sharp contrast to the thundering stroke dealt us by facts. For facts, as we have seen, while they overcome the mind, do not, as truth does, measure up to the standards of the mind as mind. Despite their bludgeoning power, they lack that clear articulation and inherent persuasiveness which appertains to intelligible objects and renders their acceptance by the mind self-imposed.

Thus, truth is its own criterion; the marks of verity rest ultimately in the clarity and nexus of our reflective insights themselves, and not in any dogmatic account of the supposed constitution of our faculties. Instead of the truth

of our apprehensions being tested by a knowledge of the psycho-physical nature of our faculties, our reliance on our faculties is rather tested by the clearness and distinctness of our theoretic insights. After all, no sense can really experience the data of another sense (sight cannot hear, or hearing see); hence our specific organs cannot ultimately test one another. It is left to the intellectual side of mind to compare the deliverances of the various natural aptitudes and to pronounce judgment upon them as regards their identity, diversity, unity, plurality, quantity, etc. Thus, when we contrast our visual impressions with those of touch or hearing, and correct the one by the other, what we rely upon are logical criteria like consistency and coherence to suggest what weight to allow the respective sense-data. For, although truth is involved in the sensuous processes of the particular thinker, it is none the less ideal in our opinion, being determined by systematic relationships which are alone objective and reveal the same structure to different minds.

Above all, it is important to distinguish the coherence that marks the formal system as true

from the "factual" or "concrete" coherence so often wrongly called by the name. In most of our efforts to get at truth, the appeal to coherence as a test is clearly present, although it is applied in a variety of ways. In daily life, for instance, it appears as the more or less crude employment of methods like those of the alibi, cross-examination, consilient testimony of independent witnesses, and reliance upon circumstantial evidence; while in science it is the guiding principle that leads the paleontologist, for example, to believe that, in excavating a set of bones which fit into one another, he is reconstructing a prehistoric animal, or the archeologist to hold that ancient inscriptions can be deciphered by patching together disconnected words (some known, others unknown) so as to "make sense." Nevertheless, there is a great difference in the way in which this coherence is interpreted by different thinkers. Exponents of the "organic" view of truth claim that an actual cohesion or linkage is observable in the physical facts themselves, no less than between facts and theories. Facts *per se* are thought to be mutually harmonious, and inclined to cleave

to each other. One event leads inevitably on to another, just as one half of a broken knife-blade fits into the other. So in the most literal way, it is believed, we can observe the world revealing a tendency to form into naturally well-rounded wholes and to fall into patterns in which the parts are reciprocally connected as means and ends.

The wide difference between a belief in "material coherence" of this sort and belief in "formal coherence" is not, of course, difficult to see; for, whereas the latter merely presupposes the subsistence of logical structure prior to experience, the former reads an immanent teleology directly into it. Since Hume, however, and the rise of evolutionary theory, not a few philosophers have become persuaded that there is no inherent purposiveness observable in experience, no positive linkage or interlocking of the parts. With this we agree. Facts themselves, in our opinion, cannot rightly be said to form a pattern; at most they appear only conjunctively associated in experience. And, because they lack adhesive power, it is absurd to imagine that the discovery of certain factual "missing links"

can ever fill up the gaps in a tentative system or hypothesis. By itself, so-called "material coherence" is insufficient to establish an hypothesis. One of the reasons for this is that facts have only negative evidential value. They may tell us that reality is inconsistent with such and such an hypothesis (as where the established presence of a man at a given place and time disproves the theory that he committed a crime at another place at the same time); but, on the other hand, no amount of facts can ever, by their apparent connivance and collusion, of themselves suffice to prove a theory. For it always remains possible that some new theoretical interpretation of the arrangement of the facts may be discovered that will place the situation in quite a different light. Hence it is that we say certitude never attaches to matters of fact. Furthermore, a simple listing of the material constituents of an entity or event, together with an empirical description of the way in which the parts apparently go together, can never assure us that we have actually in our possession all that is necessary to constitute the whole. For, after all, through oversight or malobservation,

it is possible that elements may still be lacking; and no matter how long we continue to search for them, mere experience lacks authority to end the search. Only by claiming a universal *a priori* insight into the nature of the object as a system, do we gain the right to affirm, on viewing the bones of a dinosaur or the fragments of a document, "These are all the parts there are that go to make up such an object."

It is only pure form, then, that links terms into genuinely coherent systems. And it is only as we apprehend this, and look beyond the factual content that serves as the vehicle of thought, that we lay hold of truth. Thus, the truth-value of the pieces of a knife-blade sought in connection with a murder is not in the pieces themselves, but in the total system of evidence in which they play a part. Necessary connections subsist always in a framework of reasoning, although discoverable through different contexts —here specifically through those of the properties of steel and the legal theory of crime. Moreover, when the case comes to trial, it is to be noted that the truth which we seek to infer from the consilience of circumstances surrounding the

disposition of the knife and the convergent testimony of different witnesses, is something that depends not so much upon these factual elements *per se* as upon absoluteness of the theory of probability. Thus, the reason why reliance is placed in the convergent assertions of the witnesses is that probability attests the truth of the result. If, for example, five men testify independently regarding the occurrence of the same event, and each lies half the time, the odds in favor of their "truth-telling" as a group are 31/1, since the probability of their conjoint lying is $(\frac{1}{2})^5$ or 1/32. Of course, such a definite numerical measure of the likelihood of particular witnesses' lying is not usually available; nevertheless, this does not affect the operation of the principle. Generally speaking, we can rely best upon propositions concerning events regarding which we can get multiple independent testimony (so-called "public events"); for the reason that mathematical theory indicates the high degree of likelihood of truth where there is congruence in such assertions, together with the probable cancelling out of errors.

Still other tests of truth and falsity, are ap-

parently to be found in the principle of the *reductio ad absurdum* and its complement, however difficult they may be of application. On the one hand, where the affirmation of an assertion includes within it its own denial, the assertion is said to be false. Because, for instance, in the proposition "I do not exist," there is a contradiction between the formal claim to truth of the assertion and the denial in the content of the genuineness of the subject asserted to be the source of the assertion, the statement must be false. The test of truth correlative to this is that where the denial of a statement assumes its reaffirmation, the statement is true. Thus, any denial of the truth of the assertion "There is truth" assumes the truth of the denial, and so tacitly reaffirms the original contention.

Rightly understood, the truth of anything is its explanation: the reasons that make it what it is, and the consequences that follow from it. Such explanation is, however, not the same as a genetic account, the diagnosis of a thing in terms of the history of its causes. Expositions of this sort remain essentially incomplete, always referring beyond themselves to further

facts in an endless series of empirical considerations. Some, indeed, assert that the only truth we can grasp is of this kind. Nothing, they aver, can be fully accounted for, although there appears to be no definite barrier beyond which explanation cannot go. To this, the natural question is: But if there is no bar, no hindrance at any given point in explanation, what is it that prevents things in experience from being fully elucidated? The answers usually lay the blame on human frailty or upon the endlessness of space and time. Plainly, these are lame excuses. For, if human finitude and the space-time continuum *do* finally prevent the attainment of truth, they should be frankly recognized for what they are: obstacles to comprehension that inhere radically in the very nature of things and attest to the presence in them of an element of the inexplicable. Actually, however, we do not assume or encounter anything occult or ineffable in experience. On the contrary, everywhere it seems possible to pass beyond any given limit by means of some "because" or "therefore" inherent in the nature of the field.

Nor is there any serious difficulty in the fact that no ultimate body of propositions can be found from which all others must be deduced. The failure in mathematics and in other fields to discover any set of unquestionable principles from which the rest of the propositions in that field can be systematically derived, has led certain persons to doubt the possibility of finding particular instances of truth altogether. But the error of this sort of criticism, as it seems to us, is again that of looking for *truth as fact,* the error of expecting to locate truth in the content of a specific set of propositions rather than in the formal relationship indicated by them. After all, no content as such is true; hence the search to find one is idle and bootless. Only in so far as the content is comprehended as a sign or particular expression of a general law can it be said to partake of truth.[5]

[5] If asked to cite specific examples of meanings that embody valid relations in such a way as to share their indefeasibility, we might perhaps mention the "conservation" and "indestructibility" of substance as necessary presuppositions of the phenomenal world, based upon the notion that "whatever is, is" (while "what *is not*" is excluded from its nature). Or, again, we might offer the political proposition "men are equal" as an example of what we mean by a truth. This notion too, in our opinion,

Thus, the truth of anything is that thing set in a pattern of its grounds and consequents, so that apprehension of its meaning in this context conveys an insight into its necessity. And, though no final reason for anything can be adduced in the sense of an end-term in the chain of existents, nevertheless each explanation, so far as it goes, is a closed system. Hence it is a mistake to suppose that all truth is relative, or that what is true in a narrower system becomes false in the context of a wider one. On the contrary, each truth is absolute within limits. Given certain terms and postulates, certain conclusions follow from them. In this sense, every truth is relative to certain conditions; change these, and you have a different truth to consider.

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goes back to the law of identity. Acceptance of the principle that *A* is *A*, when specifically applied to human entities, becomes the recognition that "I am I," "you are you," and that, since each is as much himself as is any other, logically speaking all are equal. For man alone among identities not merely *is himself* but *affirms that he is himself*, thus appropriating his identity as a conscious right. He does not merely come under the law but applies the law to himself. Thus, with the emergence of rational self-reference, the law of identity ceases to be merely a bloodless presupposition to which all experience must conform, and comes to function as a logical norm within experience in a totally different way.

Finally, truth is not to be sought in some particular content or formula but rather, in our opinion, in the inexhaustible fertility of inferences and the insight that inference itself is indestructible.

CHAPTER IX

THE UNIVERSE AND UNIVERSALS

I: The Claim to Universality

PHILOSOPHY, it is more or less generally agreed, seeks to understand the universe. Yet, whether there is such a unitary objective state of affairs as the universe, or, if so, in what sense there is, is a question often productive of the vaguest answers. That we do actually allude with apparent meaning in daily life to the sum-total of things, can hardly be doubted. But it may be asked whether, when we thus refer to the all-inclusive totality in the singular, we may not be deluding ourselves with a merely specious object of reflection, devoid of truth and ontological significance. There are many who in one fashion or another have urged this to be the case: some claiming that "the universe" is nothing more than a name; others, that it denotes some vague image in our minds or that it is only a supreme class-concept under which we subsume, for convenience in our thinking, the aggregate of sensuous particulars; while still others believe in the totality only as

a regulative ideal, as something which we may hope for or even "make believe" to exist, but which remains at bottom a heuristic fiction bereft of being in the full sense.

Now it is the aim of the present chapter to uphold the traditional or at least the nominally accepted view that the universe is the subject-matter of philosophy, by maintaining not only that the universe is a possible object of investigation, but also that it is the object presupposed by all others and possessed of ontological significance in the genuine meaning of the term. The natural place to begin such a discussion is, as it appears to me, with a consideration of the nature of universals; for, however we interpret the universe, most of us will be inclined to admit that it is not a strictly sensuous object, and that, therefore, if it is to be approached at all, it must be by means of concepts rather than of percepts; that is, by those collective objects of generic thinking that the logician designates as "universals." A universal may be provisionally labelled as a unitary description that covers all of a kind; and, though we may question in advance whether there can be a supreme uni-

versal equivalent to the kind of all kinds, we seem more or less obliged to assume that, if there can be an object inclusive of all others (viz., the universe), knowledge of it must presumably be arrived at by the same means as our comprehension of universals.

Reflective experience (that is, conceiving, judging, and inferring) seems to advance by laying hold of the form of objects; that is to say, whenever we think, we grasp the object of thought not merely as a particular existence, but as an illustration of a general principle. For thinking involves comparison and proceeds by the detection of likenesses and differences between things; and, wherever objects are compared, use is made of some identity or standard of commensurability whereby a bond of interrelation is disclosed among them. This common denominator employed by thought does not, however, appear to have the nature of a sensuous property shared by the particulars, but has rather the appearance of a norm assumed by reflection. If I could discover that a particular *A* was like or unlike a particular *B* only by introducing a third particular *C* between the

two, presumably I should never discover it, since I should have to compare *C* with *A* and *B* taken separately to make sure it was like or unlike each of them, an operation which would in turn require the introduction of a further standard of comparison *D*, and so on *ad indefinitum*. The conclusion seems to follow that the reference to a standard in thinking is not to some empirical item in experience but to some determinant which we bring to the world of experience. Everywhere we line up and evaluate our particular objects in the light of an absolute. Although I can never see (i.e., sense) this page as pure white, nevertheless there is that about it which makes me know what pure white is. Similarly, we judge the specific circle in the light of the idea of the perfect circle; and our act of duty is inspired by a *faciendum* that is more than any *factum*. In such instances, we appeal to a determinant which is identical neither with the individual phenomenon nor with the sum-total of existent circles or acts of dutifulness, yet which is none the less taken to be typical of all possible examples of their kinds. Thought, then, if we are not mistaken, has the power of

apprehending an ideal totality from an empirical case, of seeing the one through the other, and of passing from particulars to wholes distinct from the mere collection of the parts.

But, if thought really involves reference to a standard that holds for all cases, by means of which we may assess the value of our particular experiences, surely such a mode of apprehension, if any, must furnish us with knowledge of the universe. Indeed, it may even seem that the idea of the universal when fully unfolded is scarcely distinguishable from that of the universe. For, although the universal is clothed with different meanings in connection with different sets of objects (e.g., in the realm of circles it is the "perfect circle," in the chemistry of gases the "perfect gas," or in the sphere of human beauty the ideal of physical perfection), nevertheless throughout *many* appearances it continues as fundamentally *one*, since behind all diversities of application remains the unifying notion of a standard to which all things are referred. The universal, though comprehended under many shapes, everywhere expresses the principle of an ideal relation unifying things.

II: The Ontological Reference in Thinking

In addition to the claim to universality, thought assumes that it possesses ontological import. This is evident in its assertion of objective reference and in its alleged grasp of something "beyond" and "other than" itself. Practically all judging and inferring refer to an Other than the act of judging or inferring. I do not merely "think what I think" or "have an idea of an idea." Were reflection no more than this, it would involve an endless series of apprehensions of mental states by mental states, and be self-refuting in its attempt to refer beyond itself. There is, however, no concrete evidence in the thinking process of the existence of such an infinitely regressive pursuit of ideas by acts of ideation. Moreover, any skepticism as to the valid possibility of referring to an object distinct from our mental states would, if entertained, cast doubt upon the integrity of thinking altogether; for the genuineness of thought consists precisely in its power to *mean what it claims to mean.* Hence, any repudiation of the object of thought in general would be a chal-

lenge to the validity of the process as well. It would seem, then, that thought, on its own presuppositions which alone render it significant, must accept the authenticity of "something more" to the world than the mere act of reflection itself.

III: The Universe as the Ultimate Object of Reference

But, assuming the authenticity of the ontological reference in thinking, we have next to ask what evidence there is that this "something more" to which being is attributed is none other than the universe. The indications which point to the universe as the ultimate object involved in reflection seem to be of three sorts: first, that involved in the discovery of the inexhaustibility of particular objects, anyone of which appears, when examined, to implicate so vast an array of factors that no boundaries can be set to it short of the totality itself; secondly, recognition that what the mind finds in its object is uniformities, dependencies, serial connections, which hold beyond ascertainable limit; and, thirdly, the conclusion that objects are im-

bedded not merely in an endless causal chain or linear order but in a closed, integrated system in which the totality of the constituents presumably remains constant.

First, let us consider how an indefinitely expanding reference to the universe as the sum-total of phenomena may be involved in the act of thinking. Suppose I am reflecting upon such a simple object as this book. Though my recognition of its existence implies that the object is taken to be distinct from my act of thinking about it, yet I also recognize that the book, in so far as it lends itself to being experienced, must possess a nature which is in some sense determined by the experiencer. Yet, in defining the book in terms of what it can be experienced as, an indefinite number of physical and psychological factors come to be accounted part of it. Not only may the size and volume differ with the various standpoints of the observer and the differing space-time coördinates that he may employ, but such aspects as color are also plainly dependent upon the sensitivity of the sensory nerve-endings of the receiving organism, the wave length of light,

and an incalculably intricate interplay of conditions. In brief, if the conditions of a thing enter into it as factors, no apparent boundaries can be set to its nature. Yet, if this be true, and if no finite group of elements can exhaust the nature of a thing, we seem finally led to assume that nothing less than the totality of affairs or the universe itself can acquaint us with the thing in its complete essence.

Again, there is another sense in which thought refers beyond its occasion to the totality. This is discernible in the fact that the mind always interprets its object as an illustration of certain laws. While mere sense-data are defined by what they are relative to, and, as such, are capable of an indefinite variety of interpretations by different perceivers, the full nature of an object is to be found not in its merely qualitative character but in the framework of relations and universals which are somehow assumed to make it what it is. What the mind ultimately seeks and finds in its object apparently is uniformities, patterns, in brief, an "order" of nature. This ball, for instance, which I bounce, will be interpreted by me, in so

far as I am scientific, as an exemplification of the laws of motion and mass; and even the child who runs forward to seize it will assume that its behavior is controlled by certain fixed determinations and that, having bounced once, it will bounce again. Alike in our simplest actions and in our most complex thoughts, we presuppose that it is possible in any given experience to pass from certain phases to certain others on the basis of some lawful identity shared among them. This tendency of thought to break down the barriers of "this" and "that," and to comprehend things in their connections as examples of relationships which hold beyond ascertainable limit, is equivalent to the postulate of the lawfulness of nature. Nevertheless, the discovery of an apparently inexhaustible dependency tells us no more than that everything has an indefinitely inferable and referable character and serves as a link in a chain of endless causal connections; and, though it seems to tell us that nature is uniform and that what is once true must be true again, it does not tell us whether nature is coherent or show any reason why the linkage of cause and

effect should be more than merely a linear series or should, on the whole, form a closed order or mechanical system.

Yet this third assumption to the effect that the totality must be somehow a more or less coherent unity of interrelated parts, appears also involved in our apprehension of objects. When I think of this book in my hand, my recognition of its genuineness implies that it is what it is; in other words, that it has a nature of its own distinct from what I or any other observer determine it to have. Although man the observer may be the measure of his world of things as regards their matter, this seems not necessarily the case as regards their form, which stands opposed as an *Anstoss* or foil to the mental act. We must, at least, affirm the genuineness of the book as something which can be referred to, classified, compared, and which appears to act in the same way under the same conditions. Moreover, whatever is recognized as an entity is by the same act distinguished from other entities in a purely formal way. Thus, the basis for discriminating "this" object from "not-this" is not some qualitative diver-

gence of the one from the other, but rests in the apprehension of the logical identity of each, and of the fact that each is itself in the same sense as every other. Yet, as will appear, in this admission that the very being of objects implies their formal identity and separateness irrespective of their qualitative aspects, the assumption of coherence lies implicit.

For certainly identity, which is central in the notion of lawfulness, indicates that something is the same and unchanging: of the whole universe no less than of the parts, we are entitled to say "whatever is, is." Yet, wherever an attempt is made to learn the specific character of what is invariant, the answers seem conflicting. Though the sciences are apparently committed to the assumption that something is conserved as unchanging, there is still divergence of opinion as to exactly what it is. The prevalence of qualitative change and the relativity of appearances raise what look like insuperable barriers against the discovery of any constant nature in the sensible material of the universe. Instead, we seem forced back upon a consideration of its formal properties;

for, throughout the kaleidoscopic flux of qualities, the formal identity and plurality of entities at least remain. Laying hold of these fundamental notions as unities, science (though without any proof of actual phenomenal simplicity) has succeeded in building up elaborate atomistic pictures and hypotheses indicating ways in which phenomena in different fields can be taken as functions of one another. In other words, out of the idea of bare, almost qualityless units, thought has somehow been able to construct a system of substitutions and representations, transformations and equations, which, while leaving each entity and the whole what it is, nevertheless succeeds in suggesting how the discrete, detached appearances may be correlated with one another. By degrees, the notion of quality is subordinated to that of quantity. What remains the same is seen to be not the kind of stuff but the quantum of it; so that to the query "quantity of what?" the answer might well be "why, of that which is everything yet in itself nothing." In science, this supposition of the constancy of the whole is expressed as the principle of the conservation of

mass and energy. Only in a world governed by some such postulate of permanent relationships does it seem possible to have assurance that the balance of nature may not be upset at any moment by the intrusion of unobservable and unintelligible factors.

To assert, then, that the world is constant in quantity is to indicate that it is an interrelated system. For quantity means that which is made up of parts not differing relevantly from the whole save in being less. And, if the parts differ fundamentally only in amount, while the amount of the whole is constant, the changes that take place within it must be transformations that return into themselves by a principle of equivalence. In other words, the elements would seem to be units which, although subjected to continuous translation and substitution in terms of one another, undergo on the whole no formal alteration. This last assumption of our thought appears, therefore, to require not only that its object shall be consistent, but that it shall exhibit the characters of a closed or integrated order in which the totality of the constituents remains constant.

IV: The Ontological Status of the Universe

Having shown that a reference to the universe is involved in the act of thinking, let us ask once more whether the actual being of the universe is also implied therein. Can the mere claim to existence ever prove existence? or the mere reference to a "Beyond" establish the actuality of the Beyond referred to? In other words, is it possible from the bare abstract notion of the whole to deduce its reality?

Obviously, in the narrow sense, all these questions must be answered negatively. Mere reference, of itself, we must agree, can never prove the reality of any particular world beyond the reference. For, if once you grant that the mere *claim* to mean something is sufficient to establish the *existence* of that something, you will find yourself obliged by the same logic to admit the objective reality of every prodigious fancy for which the mind of man chooses to make the claim. On the other hand, it seems impossible to doubt the genuineness of the objective reference of meaning altogether. If knowledge is possible at all, the integrity of meaning in a

general sense (that is, our capacity upon occasion to mean what we mean, and to refer to what we intend to refer to) must be accepted. Otherwise—supposing that we never meant what we meant to mean, or never thought about what we thought we were thinking about—reflection and experience would be self-refuting.

The situation seems to place us in a dilemma. On the one side, it has to be granted that meaning cannot prove the ontological reality of any external realm in particular; yet, on the other, to deny the claims of meaning altogether is virtually to declare that all is deception and illusion. Escape from the enigma is possible, apparently, only through the recognition that, although meaning as such is unable to establish the existence of any specific object of reference, it is, nevertheless, not so impotent as regards the universal object-in-general. That is to say, it may here be necessary to revive the ontological argument and to accept its validity, at least as regards the universe as a whole. Among philosophers, Spinoza especially, it will be remembered, vigorously defended the notion that totality or "Substance," unlike all less inclusive

meanings—being that in terms of which everything can be defined yet which is itself definable in terms of nothing else—"is in itself and conceived through itself," and therefore that its essence involves existence.

That the notion of the totality or universe[1] is such a meaning, seems to be so for at least two reasons. In the first place, any attempt to deny the objectivity and necessity of the total object of reference of which the thought and the thinker are both members would inevitably appeal to the authenticity of the very object denied. Any challenge of the whole must at least presuppose the genuineness of the challenge and the challenger; that is, these must themselves be construed as an authentic totality by reference to which the genuineness of *any* totality is questioned.[2] In the second place, although,

[1] As used here, the term "universe" may be taken to mean the totality of objects about which inference can take place, together with the inferences that can take place about them.

[2] Although in the case of narrower concepts, the inconceivability of the opposite can hardly prove the existence of the object conceived of, for the reason that their essences are selected and defined by a more or less arbitrary procedure which dirempts the form from the matter, the same cannot be said of the universe, which, as the all-inclusive concept and the sum of all

as has been said, the mere conceptual claim to existence is not sufficient to prove the reality of any object in particular, yet, in the case of the universe, which is the object of thought in general, it seems impossible to doubt its genuineness without doubting that of thought itself. For, if all reference by thought to an external world distinct from the act of conception were illusory, knowledge would be false and meaningless; since in no case should we ever be thinking about the reality that we thought we were thinking about. Clearly, reflection cannot grow skeptical regarding its total object without therein repudiating the validity of the reflective process itself. In other words, the mind, by its very acceptance of thought as an instrument of discovery, binds itself *ab initio* to abide by the conditions of thinking; and, among these, recognition of the reality of the object-in-general is included as one of the conditions that make thinking possible. We may conclude, therefore, that the universe, which is the ultimate object of reference in reflection, is onto-

possible selections, cannot itself be selected, or have its "what" separated from its "that."

logically posited by thought as having the same degree of genuineness as thought itself.

V: Criticism of the Universe as a Possible Object of Reference

Of the various criticisms raised against the concept of the universe and its delineation of an objective state of affairs, only the briefest notice can be taken. To allude to the universe, it is said in certain quarters, is to refer to something with no definite meaning. *Omnis determinatio negatio est;* whereas determination itself is lost when we try to conceive the sum-total of objects without the exclusion or negation of a single item. Even granting that the notion of a *summum genus* covers everything, it discovers nothing. Though by definition all-inclusive, as to the nature of what is included it affords no inkling. "Chaos" or "nonentity" so named, it is urged, would enjoy exactly as much unity and definiteness of meaning as the "universe" or "cosmos."

Yet objections of this sort, in our opinion, overshoot the mark. It is absurd to suggest that the terms "chaos" and "cosmos" are syno-

nyms or equivalent descriptions of the world. In fact, an absolute chaos, if meant to stand for an aggregate of disparates totally devoid of common characters, would seem to be inconceivable. Even the most radical empiricist will usually admit this on empirical grounds. If all entities in our world were surds or incompatibles, there could obviously be no classes, groups, or laws. Yet, if nothing of the same kind occurred twice and if nothing resembled anything else, neither could organisms, apparently, have adapted themselves to the environment nor could life have survived. Furthermore, the hypothesis of absolute chaos must be discarded on logical grounds; for even the most rudimentary conception of the mind requires some element of identity, some common essence, that runs through the particulars like a thread and unites them. In the notion of the chaos, however, the common quality that is held to be shared among the particulars is precisely the *absence of all common properties,* so that the concept, in paradoxically assuming as its connecting link the absence of all linkages, falls into self-contradiction. The idea of the cosmos,

on the other hand, is free from such apparent discrepancy; consistency and potential interrelations are characters which differentiate it from chaos. Presumably, therefore, it would seem a more adequate term to describe the world, since it at least allows for the possibility of a coherent unity of the aggregate.

There remains to be met, however, a more serious charge against the universe as a possible object of reflection: the imputation of self-contradiction and illegitimate totality. To refer to the universe as the sum of all possible items, it is sometimes said, implies an inconsistency, since the very notion of the whole itself constitutes an item not included in the whole referred to. Because of the singular fact that the thought-act is always one step beyond the object of thought, the universe can never be envisaged; and to imagine the contrary is simply to be self-refuting. Moreover, the attempt of thought to complete its incompletion generates a hopeless pursuit of totality. Thus, it is argued, any proposition that is uttered about the universe as the sum-total of facts (N) must be itself a further fact $(N + 1)$ not included in

the world to which allusion is made *(N)*. Hence, a second proposition about the universe, that shall include the first proposition, becomes necessary; but this in turn calls for another; and so an endless series is generated.

Under one form or another, this denial of the self-transcendent power of thought to embrace a totality while remaining a member within it, has always been a chief criticism of reasoning. And, in a sense, the only answer to it is simply to point out that the removal of the prerogative of self-transcendence would destroy all that makes thinking possible. Even the most elementary concept can hardly be evolved without making use of it, as may be shown by examples. For instance, the proper name "Croesus" we may suppose, at first, to have indicated a particular individual; later on, however, observation of similar qualities in others led to the name's being taken not merely denotatively as that of a single object, but connotatively as a beginning class or name typical of a class. As the characteristics of Croesus were more and more generalized, the class represented by the name became wider, yet without ceasing to in-

clude the original Croesus as a member. Yet, at this point, we may imagine that certain proponents of the theory of types, if they were consistent, might object that the definition of a totality cannot be included in the totality. They might protest that there is precisely as much contradiction in claiming that the particular individual Croesus is a member of the class of Croesuses, as there is in claiming that the act of conceiving the universe is comprehended in the universe. That is, if this view be correct, the original Croesus, whose name and individuality suffice to define denotatively all rich men as Croesuses, cannot himself be comprised in assertions about the totality of Croesuses, any more than the term "universe" can be included in assertions about the universe as an all-inclusive object; so that, if we wish to be strictly accurate and to avoid contradictions, we have to say, "I affirm what I affirm regarding all Croesuses—except the original Croesus," just as I refer to the universe as all-inclusive—except that I recognize that it cannot include my reference to it.

But surely the charge of contradiction, when

made so sweeping as to embrace all wholes and propositions that include themselves in their own scope, loses its force. Were this alone enough to constitute a contradiction, there would be few totalities or propositions true within the meaning of their own terms. The very statement, for instance, that "There can be no true assertion concerning a collection of propositions, if the proposition asserted is itself one of the collection," makes a statement covering all propositions, and hence covering itself; and, therefore, on its own terms (according to the view we are criticizing), is viciously circular and false. Thus, the original charge that any assertion about the universe involves a contradiction, can at least be met with the *tu quoque* that the very charge of contradiction contradicts itself.

In conclusion, a more positive suggestion may be made. A universal class or whole, we may urge, implies potentially at least its embodiment in an infinity of members; that is to say, the universal "justice" covers an infinity of just acts, and so on. Moreover, an infinite class or collection of objects has the peculiar property

of freedom from the axiom of whole and part: with it the part need not be less than, but may be equal to or coextensive with, the whole in which it is included. Acceptance of this notion is familiar in mathematics, where the series of squares, for instance, though forming only *part* of the series of natural whole numbers, is none the less taken to be coextensive with and equal to the number of numbers in the *whole* series of integers, since it can be placed in one-to-one correspondence with them. Now, if we assume that no contradiction is to be found in the fact that, in infinite collections, the part is taken as equal to and capable of embracing the whole in which it is contained, and if we further suppose that universals are infinite and inexhaustible in their applications, there would seem to be no difficulty in conceiving that universal classes or wholes may also without inconsistency be members of themselves and included in their own scope. In other words, granting that in infinite collections the part may encompass the whole without contradiction, this principle would seem to apply equally well to universals, permitting them without imputation of inconsistency to be

understood to stand both for a type (e.g., the universe) and for the individual falling within it.

But, undeniably, some among those who agree that all thinking involves reference to the universe, will still find difficulty in explaining to their satisfaction how, by means of universals, we actually can know the universe. Whereas the nature of the universal is abstract and general, they will say that that of the universe is concrete and singular. How, then, is it possible, by means of non-particular, abstract objects like universals, to come to know a concrete, individual object like the universe? To answer, as we have done here, that such a synthesis is possible because it is the very nature of the one to grasp the other, and because the opposite assumption (that concepts never in any case apprehend their object) would be suicidal—to reply thus we hope may not seem an over-easy solution. Like most ultimate answers, it is of the "this or nothing" type, aiming to force assent to a certain view by representing it as the sole alternative to complete intellectual nihilism. But even those who suspect the rigor

of the dichotomy as here stated may nevertheless be led to recognize the impracticality of assuming that the universals of thought are incapable of mastering the universe. Such an assumption would seem to fly in the face of too much experience. Does not science presuppose a world indefinitely investigable, to the number of whose laws no limits can be set? Does not almost every new acquisition of knowledge bear witness to some successful fusion of the abstract with the concrete? Surely to the modern mind, which constantly beholds how seemingly adamantine facts yield to the power of intellectual analysis, how apparently simple elements turn out to be complex, and how hitherto undreamt-of worlds are brought within the circle of knowledge, any claim that the universe is in its nature inaccessible to thought must be somewhat skeptically received. Thus, although it may be true that in most particular cases thought fails to exhaust the concrete individual because of the latter's infinite complexity of aspects, nevertheless we must suppose that, on the whole and in respect of the universe, the universal comprehends its object, for the reason

that it too, as we have tried to show, is provided with the same element of infinity that characterizes the whole itself.

VI: The Universe as an Object of Reason

But, if philosophy is concerned with the universe and universals, it is evident that its object is not one open to mere sensuous observation. On the contrary, such a subject as the universe is precluded from empirical apprehension not only because of the vast discrepancy between our sense-organs and the phenomena, but for the more fundamental reason that the whole essentially involves an ideal pattern, and, as such, is sharply distinguished from the sum of its sensuous parts.

In this sense, both the universe and universals are non-empirical objects. Such terms as "man," "redness," or "number," for instance, if they really convey a unitary notion of *any* or *all* of a kind, must stand for something more than shorthand descriptions of fact. Although it may be agreed that, historically, these *generalia* are elicited from experience by separating out certain common features from the sensuous

manifold, nevertheless any appeal to them as universals in thought and action presupposes that they possess an authority different in kind from that of sense-experience. Thus, concepts are rarely merely retrospective in their significance, but claim validity as an interpretation of the future—a future unfettered by empirical limits, and taken as the revelation of the determination of all possible particulars of a kind. The universal "man," for instance, must include, besides milliards in the forgotten past, human hosts yet unborn. Certainly we cannot by any device of empirical examination exhaust that indefinite sequence of humanity in the space-time order, nor can we even say that it will ever be completed. Yet we can refer to "man" significantly in the same way that we can indicate "all the rational whole numbers," even though no one will ever presumably have empirical acquaintance with more than the merest fraction of them. What these examples illustrate is that reflection has the power to grasp its object synthetically as a logical unity, without knowing the components individually and exhaustively. Thus, in passing from the

distributive to the collective significance of a group, thought exchanges the notion of a finite succession of empirical instances for that of a common essence or pervasive unity, something that can never be reached by mere counting or acquaintance with particulars. Were it not, indeed, for this element of universality in reflection, enabling it to go beyond the actual cases examined, synthetic judgments, hypotheses, predictions, and intellectually controlled advance could hardly be justified. By the same means, we may assume, the universe (though empirically inexhaustible) may none the less be grasped as a logical unity. For, if thought is able to reveal the universal in the case of lesser kinds, it cannot be difficult to understand; indeed, we are led by a continuous process to assume, that the universe itself, as the kind of all kinds, can be apprehended in the same manner.

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